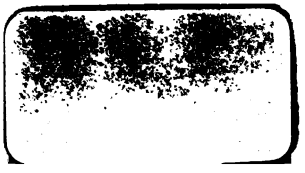

ARITHMETIC:

FOR

THE USE OF SCHOOLS.

WITH NUMEROUS EXAMPLES.

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ARITHMETIC.

INTRODUCTION.

I. ARITHMETIC, derived from a Greek word, signifying a number, teaches the knowledge and use of numbers. Numbers are of two kinds—whole numbers, and fractions. Whole numbers are such as, one, five, ten, a hundred, a thousand. Fractions (or broken numbers) are such as, a half, a third, a fourth, or three fourths. Of these fractions we shall have something to say by and by, but now we will attend only to whole numbers.

II. A little consideration will show that we always count by tens, and the reason of this is, that there are ten fingers on the two hands; the two thumbs being called fingers; from this circumstance, the first ten numbers are called digits, digit being taken from a Latin word signifying finger. The names of the first ten numbers are One, Two, Three, Four, Five, Six, Seven, Eight, Nine, Ten. The names of the next two are Eleven and Twelve, which mean, leave one, and two leave; for if you had to count eleven pebbles, you would first count ten, and leave one; if there were twelve pebbles, you would, after counting ten, leave two. It is from the Latin word for pebble that we have our word calculate, for *calculus* is the Latin for a pebble.

Next to twelve comes thirteen, then fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty. Thirteen means three and ten; fourteen, four and ten; fifteen, five and ten, and so on; twenty means twice ten; after twenty, we say twenty-one, twenty-two, or twenty and one, twenty and two, and thus to twenty-nine, and the next number is thirty, or thrice ten. After thirty we count as we did from twenty, till we come to forty, or four tens; then to fifty, or five tens; and so on to a

hundred, which is the name for ten tens. We might go further, but you ought now to see clearly that all this counting is done by tens, and the same, you may believe, is true for numbers greater than a hundred.

III. Instead of writing words for numbers, figures are used.

Thus, 1 for one, 2 for two, 3 for three, 4 for four, 5 for five, 6 for six, 7 for seven, 8 for eight, 9 for nine; besides these figures, there is the cypher 0, which is of itself of no value; but when placed on the right-hand side of a number, it increases its value in a tenfold degree.

Thus, for twenty we write 20
thirty . . . 30
forty . . . 40

Any figure placed on the right-hand of another figure, also increases the latter, in a tenfold degree.

Thus, 3 stands for three,
but 35 . . . for thirty-five;

or, by placing the 5 after the 3, 3 becomes thirty.

With the nine digits and the cypher any number may be written, though in speaking of the numbers, we must use their names.

IV. Numbers have names according to the number of figures used in expressing them; thus, numbers consisting of

One figure	are called	Units.
Two figures	. . .	Tens.
Three figures	. . .	Hundreds.
Four figures	. . .	Thousands.
Five figures	. . .	Tens of Thousands.
Six figures	. . .	Hundreds of Thousands.
Seven figures	. . .	Millions.
Eight figures	. . .	Tens of Millions.
Nine figures	. . .	Hundreds of Millions.
Ten figures	. . .	Thousands of Millions.
Eleven figures	. . .	Tens of Thousands of Millions.
Twelve figures	. . .	Hundreds of Thousands of Millions.
Thirteen figures.	. . .	Billions.

and so on, the next period being Trillions, then Quadrillions, Quintillions, &c., each period consisting of six figures, the highest denomination being hundreds of thousands.*

V. As examples, we will begin by the simple unit,

1	is	One, unit.
10	..	Ten.
100	..	Hundred.
1,000	..	Thousand.
10,000	..	Ten Thousand.
100,000	..	Hundred Thousand.
1,000,000	..	Million.

It is usual and convenient to divide numbers by commas, into periods of three, beginning from the right hand.

As examples of numbers in which other figures are used,

5 is Five.
52 .. Fifty-two.
526 .. Five Hundred and Twenty-six.
5,267 .. Five Thousand Two Hundred and Sixty-seven.
52,678 .. Fifty-two Thousand Six Hundred and Seventy-eight.
526,783 .. Five Hundred and Twenty-six Thousand Seven Hundred and Eighty-three.
5,267,834 .. Five Millions Two Hundred and Sixty-seven Thousand Eight Hundred and Thirty-four.

EXAMPLES.—What are the numbers expressed by 23; 324; 457; 1,678; 7,562; 28,931; 57,834; 783,981; 278,405; 4,001; 402,105; 7,802,391?

Write the figures for the numbers Twenty-Eight; Three Hundred and Sixty-seven; Four Thousand and Forty-five; Seventy-two Thousand and Eighty-one;

* Some Arithmeticians divide numbers into periods of three, and call thousands of millions, billions, and thousands of billions trillions.

Eight Hundred and Ninety-three Thousand Seven Hundred and Sixty-eight; Fifteen Millions; Four Millions Five Hundred and Sixty-seven Thousand Eight Hundred and Ninety-one.

VI. The examples here given will be deemed sufficient, since children, before they come to the use of arithmetic, are taught by the black board, or in some other way, the mode of writing down in figures any given number, or of reading any given collection of figures. The former case, of writing down figures, is called Notation; the latter, of expressing the symbols of figures in words, is called Numeration.

The annexed is the usual form of what is called

THE NUMERATION TABLE.

8	9	7,	6	1	3,	4	5	6,	7	2	3,	4	5	6
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Thousands of Millions.	Tens of Thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.

&c.

VII. Having now described the method by which any given number may be expressed, or any collection of figures may be read, we resume the definition of Arithmetic, which is the science or knowledge of numbers. It demonstrates the truth of rules, which enable the mind to perform calculations otherwise very fatiguing, and to many persons, without such aid, practically impossible.

VIII. All the rules of Arithmetic may be referred to four primary rules, viz :—

Addition : Subtraction : Multiplication : Division.

Addition teaches how various numbers may be collected into one number, called their sum.

Subtraction teaches us to find the difference between any two numbers.

Multiplication teaches us to perform at one operation the repeated additions of the same number.

Division teaches us to perform at one operation the repeated subtractions of the same number, from another given number.

All other rules in Arithmetic are applications of some one or other, or of all, of these rules.

It may be observed, that in fact there are but two rules, Addition and Subtraction : for Multiplication is only a compendious (short) method of Addition, and Division a short method of Subtraction.

IX. We have said that numbers may be divided into two kinds,—whole numbers, also called integers, and fractions. But numbers, whether they be integral or fractional, are also called abstract or concrete.

An abstract number is when the symbol of the number is only written down, or its name pronounced, as 5, 7, or five, seven, without any reference to things ; a concrete number is when the number expresses a certain collection of things, as 2 pounds, 5 horses, 7 trees.

The idea of number independent of quantity is perhaps mentally impossible ; we tacitly refer to something as a unit : and the number indicates the multiple of the unit, or the times that the unit is repeated. Thus 7 conveys by itself no idea, till we think of 7 inches, 7 pounds, 7 apples, or 7 pebbles ; the unit being an inch, a pound, an apple, or a pebble.

In the Four Rules called the Simple Rules, we keep the unit out of sight ; but in the first instance, children are best taught by visible representations of number.

X. The following signs are made use of in Arithmetic :—

+ (read *plus*), the sign of Addition, shows that the number placed after it is to be added. Thus, $5 + 3$, means that 3 is to be added to 5.

— (read *minus*), the sign of subtraction, shows that the number placed after it is to be subtracted; thus, $5 - 3$, means that 3 is to be taken from 5.

× the sign of Multiplication, (often read *into*,) signifies, when placed between two numbers, that their product is to be found; thus, 5×3, is read 5 into 3, or, find the product of 5 and 3.

÷ the sign of Division, placed between two numbers, indicates that the former is to be divided by the latter; thus, $12 \div 2$, may be read 12 divided by 2.

= the sign of equality, shows that two numerical expressions are equal; thus, $5 + 3 = 8$, which may be read, 5 plus 3 equals 8.

: the sign of comparison or ratio; thus, 5 : 7 is read, as 5 is to 7, and means that five things are compared with seven things.

: :: : the sign of proportion. This is explained in the Chapter on Proportion.

∴ the sign for therefore.

CHAPTER I.

THE SIMPLE RULES.

ADDITION.

XI. ADDITION is the method of collecting together several numbers into one number, called the sum.

Since every sum in Addition is only a continual repetition of the addition of two numbers, the class should be first well exercised in questions of the following kind:—

What is the sum of	3 and 5?	of 2 and 7?
„	of 6 and 4?	of 3 and 6?
„	of 8 and 7?	of 6 and 5?
„	of 9 and 3?	of 8 and 6?

This being done, the rule for the addition of numbers, is, First write the numbers under each other, units under units, tens under tens, hundreds under hundreds, and draw a line under them; then add the digits in each column, beginning with the units, into one sum, and write the result underneath.

Ex. (1.) Find the sum of 324 and 241.

$$\begin{array}{r} 324 \\ 241 \\ \hline 565 \\ \hline \end{array}$$

Here add the units column first, and 1 and 4 make 5; in the tens column, 4 and 2 make 6; and in the hundreds column, 2 and three make 5; and the sum is 565.

The same result would be obtained had we first added 200, then 40, and then 1. For by adding 2 to 3, each being in the hundreds column, we do add 200 to 300, making it 500; and by adding 4 to 2 in the tens column,

we add 40 to 20, making it 60; while in the units column, the addition of 1 to 4, makes 5 units.

Ex. (2.) Find the sum of 237 and 956.

$$\begin{array}{r} 237 \\ 956 \\ \hline 1193 \\ \hline \end{array}$$

Here, adding the units column, 6 and 7 make 13, set down the 3 and carry the 1, which is 10, to the tens column; and say 1 and 5 make 6, and 3 make 9: here there is nothing to carry, since the sum is less than 10; now add 9 to 2, which makes 11, the whole of which is written down, since we have no more columns to add.

Examples.

231	472	72	45	7912	23489
142	516	87	67	569	37845

The proof of these simple sums is by adding the top line to the lower line.

Ex. (3.) Find the sum of 534, 682, 387, 573, 126, 934. Write the numbers directly under each other.

$$\begin{array}{rr} & 534 \\ & 682 \\ & 387 \\ & 573 \\ & 126 \\ & 934 \\ \hline \text{Sum} & 3236 \\ \hline & 2702 \quad \text{Sum omitting top line.} \\ \hline \text{Proof} & 3236 \\ \hline \end{array}$$

The sum of the units column is 26, set down 6, and carry 2 to the tens line; the sum of this with 2 is 33; write 3 and carry 3 to the hundreds column: this makes the sum of that column 32; write down the 2, and

since there is nothing to add to the 3, put it on the left hand of the former three numbers, and the sum is 3,236.

The best method of proof, is by re-adding the figures, from the top; or we may draw a line under 534, the upper number, then find the sum of the remaining numbers, or 2,702; add to this 534, and if 2,702 and 534 make the former sum, or 3,236, the work is correct.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
41	27	29	56	88	45	23
23	75	37	65	90	39	75
72	69	46	37	11	41	37
68	54	58	99	13	34	65
46	82	62	11	97	23	90
35	33	73	40	49	78	39
53	77	84	57	23	97	68

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
436	392	735	881	702	817
412	659	685	119	537	972
729	397	458	723	784	343
568	269	991	353	572	439
845	460	334	373	397	731
237	371	468	404	783	889
537	141	262	978	419	942

(14.)	(15.)	(16.)	(17.)	(18.)
2147	3147	5858	3957	8974
5825	6695	1482	5374	3384
3346	9790	9433	1897	2919
2129	1164	7112	8374	3289
8577	9342	5707	2739	5753
6234	6639	8433	9347	4047
5937	9631	3919	3875	1234

(19.)	(20.)	(21.)	(22.)	(23.)
9134	2810	8563	2138	9463
7754	6294	3662	8454	2345
2555	4241	3539	5693	2377
9420	6134	7452	6725	5928
6329	2024	8846	9862	6892
6437	4548	6724	1868	9890
1038	1275	9483	3925	3702
8812	8023	2239	7392	3948

(24.)	(25.)	(26.)	(27.)	(28.)
41263	67890	47299	25825	41448
75991	45486	49788	34835	53274
99624	72418	93349	92662	23491
15728	68980	22596	73196	82210
21093	37023	93325	58293	52081
50412	73923	82534	46245	42642
36722	94822	83592	63542	35522
12345	87536	66273	83247	33160

(29.)	(30.)	(31.)	(32.)	(33.)
77546	82291	33458	73409	54238
436	48	25437	63157	6
2845	22835	52	746	45793
45	34	94364	62452	45
9661	23521	84592	74238	69849
88043	35684	87	330	58909
34	38	74538	22553	27
45097	32572	756	25	63798
2646	439	83540	19432	83
62418	97883	8784	84525	22524

(34.) At the census taken March 31, 1851, the population of England and Wales was found to be 17,922,768 persons; of Scotland, 2,870,784 persons; of Ireland,

6,515,794 persons; of the islands in the British seas, 142,916 persons: find the total population of Great Britain and Ireland.

(35.) The number of visits to the Great Exhibition was—in May, 734,782; in June, 1,133,166; in July, 1,314,176; in August, 1,023,435; in September, 1,155,240; in October, 841,107: find the whole number of visits.

(36.) Six bags of nuts were computed to contain respectively 2538, 7290, 6183, 2745, 8324, and 7777 nuts: How many were there in all?

(37.) Adam lived 930 years; Seth, 912 years; Enos, 905 years; Cainan, 910 years; Mahalaleel, 895 years; Jared, 962 years; Enoch, 365 years; Methuselah, 969 years; Lamech, 777 years; Noah, 950 years: find the total number of years that the first ten patriarchs lived.

(38.) How many days are there from May 1 to Oct. 11, both days inclusive?

(39.) How many days are there in the 5 years beginning January 1, 1848, and ending December 31, 1852.

SUBTRACTION.

XII. SUBTRACTION teaches us to find the difference between a greater and a less number: the latter being taken from the former.

The difference is called the remainder.

The class should be exercised *vivâ voce* in the subtraction of the digits from each other. Thus: Take 3 from 7: what is the remainder? Take 5 from 9: what is the remainder?

Rule 1.—Place the less number under the greater: units under units; tens under tens; and draw a line below them.

2.—Take the units digit in the lower line from that in the upper line, and beneath write the remainder. Then take the tens digit of the lower line from the corresponding one in the upper, and write down the remainder; and so on.

3.—But if the figure in the lower line be greater than that immediately above it, add ten to the upper one, and take the lower figure from the sum, and write down the remainder.

4.—Carry one to the next lower figure; and proceed as before, till all the figures are subtracted.

To prove, add the remainder to the less number, and then the sum must be equal to the top line, or greater number.

Ex.—Subtract 234 from 876; and 287 from 725.

	(1.)		(2.)
	876		725
	234		287
Remainder	642	Remainder	438
Proof	876	Proof	725

In (1.) each figure of the lower line is less than the corresponding figure in the upper, and the remainder is easily found to be 642. To prove this, add 234 and 642 together, and their sum is 876, which is the top line, or greater number. The same result may be obtained by subtracting from 876: first 200, then 30, and then 4.

In (2.) 7 being greater than 5, we add 10 to 5, which makes 15; then 7 from 15 leaves 8; write 8 down; carry 1 to 8 in the tens place of the lower line, by which we return the 10 we borrowed. 1 and 8 make 9; 9 from 2, we cannot; borrow 10; 10 and 2 make 12; 9 from 12 leaves 3; write down 3. Carry the borrowed 1 to 2, which makes 3; 3 from 7 leaves 4: and the remainder is 438. This remainder, added to 287, makes 725; which proves the accuracy of the work.

Examples.

(1.)	(2.)	(3.)	(4.)	(5.)
274	489	285	4625	89723
123	273	167	2737	59845

(6.)	(7.)	(8.)
5287632	72435689	2682473453
4937843	25823492	748392624

(9.)	(10.)	(11.)
458003214	5003000201	9786543201
279335162	1234567892	1987654321

(12.) William I. began to reign over England in the year 1066; Queen Victoria began to reign 1837: What is the number of years between the conquest of William and the accession of Victoria?

(13.) The year of our Lord's birth is said to be 4004 years after the year of the world, or 4004 A.M.: find the years before Christ of the following events—Deluge, 1656 A.M.; Call of Abraham, 2083, A.M.; Departure from Egypt, 2513, A.M.; Building of Solomon's Temple, 3000, A.M.; Isaiah's Prophecy of the Messiah's Coming, 3262 A.M.; the Babylonish Captivity, 3416, A.M.

(14.) A horse, with his saddle and bridle, is worth 63*l.*; but his bridle and saddle cost 8*l.*: What is the price of the horse?

(15.) Out of a bag of 1000 sovereigns, I took 150, then 65, then 275, and lastly 287: How many were left?

(16.) The population of England and Wales was, in 1801, 8,892,536, and in 1851, 17,922,768: find the increase in the fifty years between 1801 and 1851.

(17.) The population of Scotland, in 1801, was 1,608,420; in 1851 it was 2,870,784: find the increase in the fifty years between.

(18.) The population of Ireland was, in 1841, 8,175,124; in 1851 it was 6,515,794: find the decrease in ten years.

(19.) At the Great Exhibition, the total receipts were 505,107*l.* Of this sum the subscriptions were 67,399*l.*; season tickets, 67,610*l.*; profit by refreshments, catalogues, &c. 13,290*l.*: find the remainder, which was the receipt at the doors.

(20.) America was discovered in 1492; the Reformation began 1517: How many years are there between these events and 1852?

(21.) How much is fifteen millions two hundred and four, greater than a million and a half and four hundred and two?

(22.) In 1812 the first steam-boat was propelled on the rivers of America: How many years is it since?

MULTIPLICATION.

XIII. To multiply one number by another number, is to find the number made by repeating the first number as many times as there are units in the second number.

Thus, to multiply 25 by 6, means that the sum of 25 six times repeated is to be found, as in the margin.

25
25
25
25
25
25
150

Instead, however, of using repeated additions, we can, by means of a table, called the Multiplication Table, obtain the result in a very expeditious manner. It is first, however, necessary that the multiplication table should be thoroughly committed to memory; and no step should be taken in advance until this is completely effected.

The number to be multiplied is called the Multiplicand.

The number by which you multiply is called the Multiplier.

The result is called the Product.

Sometimes the multiplicand and multiplier are called Factors.

We now give the Multiplication Table, the early portions of which are well exemplified by counters.

MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 — 4	2 — 6	2 — 8	2 — 10	2 — 12	2 — 14
3 — 6	3 — 9	3 — 12	3 — 15	3 — 18	3 — 21
4 — 8	4 — 12	4 — 16	4 — 20	4 — 24	4 — 28
5 — 10	5 — 15	5 — 20	5 — 25	5 — 30	5 — 35
6 — 12	6 — 18	6 — 24	6 — 30	6 — 36	6 — 42
7 — 14	7 — 21	7 — 28	7 — 35	7 — 42	7 — 49
8 — 16	8 — 24	8 — 32	8 — 40	8 — 48	8 — 56
9 — 18	9 — 27	9 — 36	9 — 45	9 — 54	9 — 63
10 — 20	10 — 30	10 — 40	10 — 50	10 — 60	10 — 70
11 — 22	11 — 33	11 — 44	11 — 55	11 — 66	11 — 77
12 — 24	12 — 36	12 — 48	12 — 60	12 — 72	12 — 84

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 — 16	2 — 18	2 — 20	2 — 22	2 — 24
3 — 24	3 — 27	3 — 30	3 — 33	3 — 36
4 — 32	4 — 36	4 — 40	4 — 44	4 — 48
5 — 40	5 — 45	5 — 50	5 — 55	5 — 60
6 — 48	6 — 54	6 — 60	6 — 66	6 — 72
7 — 56	7 — 63	7 — 70	7 — 77	7 — 84
8 — 64	8 — 72	8 — 80	8 — 88	8 — 96
9 — 72	9 — 81	9 — 90	9 — 99	9 — 108
10 — 80	10 — 90	10 — 100	10 — 110	10 — 120
11 — 88	11 — 99	11 — 110	11 — 121	11 — 132
12 — 96	12 — 108	12 — 120	12 — 132	12 — 144

XIV. When the multiplier does not exceed 12:

Rule 1.—Write down the multiplier under the last figure or figures on the right hand of the multiplicand, and draw a line under them.

2.—Multiply every figure of the multiplicand by the multiplier, and in each successive product write down the remainder above tens under the figure you have multiplied.

3.—Carry the tens considered as units to the product of the next figure, till every figure of the multiplicand has been multiplied. The result is the product required.

Multiply 84367542 by 6.

84367542
6
506205252

We see 6 times 2 are 12; write 2 and carry 1. 6 times 4 are 24, and 1 make 25; 5 and carry 2. 6 times 5 are 30, and 2 are 32; 2 and carry 3. 6 times 7 are 42, and 3 make 45; 5 and carry 4. 6 times 6 are 36, and 4 make 40; 0 and carry 4. 6 times 3 are 18, and 4 are 22; 2 and carry 2. 6 times 4 are 24, and 2 are 26; 6 and carry 2. 6 times 8 are 48, and 2 are 50; which is written down.

EXAMPLES.

	Multiply		Multiply
(1.)	23456789 by 2	(2.)	479083456 by 2
(3.)	73624345 by 2	(4.)	362500799 by 2
(5.)	56789736 by 3	(6.)	392592483 by 3
(7.)	48319476 by 3	(8.)	627199016 by 3
(9.)	53189447 by 4	(10.)	925924836 by 4
(11.)	36243457 by 5	(12.)	8973604831 by 5
(13.)	61150683 by 6	(14.)	1248649024 by 6
(15.)	435000891 by 7	(16.)	7647194132 by 7
(17.)	79170508 by 8	(18.)	360440763 by 8
(19.)	483194672 by 8	(20.)	2596048317 by 8
(21.)	312567835 by 9	(22.)	2719927352 by 9
(23.)	478753626 by 10	(24)	8976342500 by 10
(25.)	531362434 by 11	(26.)	2345678941 by 11
(27.)	362434531 by 12	(28.)	7652230745 by 12
(29.)	5318973456 by 12	(30.)	3623921997 by 12

(31.) In 9 boxes of oranges, each containing 12 dozen oranges, one bad in every dozen; how many sound oranges?

(32.) In 11 boxes, each containing 356 sovereigns and 1372 shillings, how many sovereigns and how many shillings? and find how many coins there are altogether.

(33.) In the 10 years beginning January 1, 1843, and ending December 31, 1852, how many days?

XV. When the multiplier is greater than 12:

Rule 1.—Write the multiplier under the multiplicand, units under units, tens under tens, &c.

2.—Multiply the multiplicand by the figure in the units place, as in the preceding rule.

3.—Then multiply the multiplicand by the digit in the tens place of the multiplier, placing the first figure of the product under the tens place of the previous product; and so proceed with the other figures of the multiplier.

The sum of the separate products will be the whole product required.

Ex. Multiply 2783456 by 37.

2783456
37

19484192
8350368

102987872

In fact, we first multiply the multiplicand by 7, and then by 30: the cypher which would result from the latter multiplication is omitted; it would, if put down, be under the 2 in the units place of the first multiplication, and the sum of these products will be the whole product of 2783456 multiplied by 37.

XVI. When the multiplier has cyphers on the right-hand side, the first significant figure of the multiplier is placed under the unit of the multiplicand, and the cyphers are brought down.

Multiply 57832 by 2000.

57832
2000

115664000

XVII. When both the multiplicand and multiplier have cyphers on the right hand, multiply the significant figures as in ordinary multiplication, and add to the result the sum of the cyphers in the multiplier and multiplicand.

Multiply 525000 by 2300,

525000
2300

1575
1050

1207500000

EXAMPLES.

Multiply	Multiply
(34.) 796456783 by 13	(35.) 571243624 by 17
(36.) 710443697 by 19	(37.) 345786456 by 23
(38.) 815096582 by 37	(39.) 294685712 by 43
(40.) 728145734 by 65	(41.) 546744839 by 58
(42.) 914683791 by 79	(43.) 786456294 by 97
(44.) 256374892 by 88	(45.) 467583925 by 59
(46.) 398764532 by 345	(47.) 212685712 by 507
(48.) 603820409 by 376	(49.) 946858397 by 365
(50.) 146857912 by 824	(51.) 285690518 by 907
(52.) 361807216 by 925	(53.) 564397683 by 782
(54.) 154276488 by 968	(55.) 736618597 by 653
(56.) 914736185 by 678	(57.) 453383794 by 387
(58.) 86957203 by 1234	(59.) 28573962 by 2468
(60.) 14736285 by 3791	(61.) 46858394 by 5636
(62.) 61605995 by 9758	(63.) 79653427 by 7354
(64.) 43051798 by 2976	(65.) 56780932 by 6688
(66.) 79375486 by 7052	(67.) 68374589 by 8009
(68.) 27394658 by 4963	(69.) 46758571 by 9867
(70.) 61807125 by 5702	(71.) 96583400 by 5904

	Multiply		Multiply
(72.)	27658000 by 3500	(73.)	45937200 by 9700
(74.)	194682372 by 23564	(75.)	28578396 by 41789
(76.)	23456789 by 56237	(77.)	68375581 by 77839
(78.)	45678900 by 20000	(79.)	457329000 by 7000

DIVISION.

XVIII. To divide one number by another is to find how many times the former is contained in the latter.

The operation is called Division.

The number to be divided is called the Dividend.

The number by which we divide is called the Divisor.

The number of times it is contained in the Dividend is called the Quotient.

If the number be not contained an exact number of times, the number left is called the Remainder.

XIX. Thus, to divide 12 by 4, we ask how many 4s in 12; then remembering that $3 \times 4 = 12$, we say 3: here 12 is the dividend, 4 is the divisor, and 3 is the quotient, and there is no remainder. But to divide 14 by 4, we know that $4 \times 3 = 12$, and $4 \times 4 = 16$, so that there are 3 but not 4 fours contained in 14; and as 14 exceeds 12 by 2, we say that 4 is contained in 14, 3 times, with remainder 2.

We should obtain the same results, had we successively subtracted 4 three times from 12 or 14; in fact, Division, as we have before said, is a short way of doing Subtraction, or we may say that division is the rule by which, having giving a number and one of the factors of which it is composed, we find the other factor or the number nearest to the other factor.

XX. Hence Division may be taught by the multiplication table, and it is, in fact, the best way of making the young pupil expert in both multiplication and in division.

Thus, since $7 \times 8 = 56$, how many 7s are there in 56?
„ $9 \times 8 = 72$, how many 9s in 72?

Next, how many 9s are there in 85 ?

Since $9 \times 9 = 81$, and $9 \times 10 = 90$, there is not an exact number of 9s in 85 ; there are nine, but not ten; and, since 81 taken from 85 leaves 4, we say that there are 9 nines, with remainder 4.

XXI. *Rule* 1.—When the divisor does not exceed 12 (called Short Division), write down the dividend, and draw a curved line on the left hand, and a straight line under it. Put the divisor on the left hand.

2.—Find how many times the divisor is contained in the first, or the first two figures of the dividend ; and write down the quotient under the first figure, if the divisor be less than it ; but the second figure, if the divisor be greater than the first figure.

3.—Then, to the right of the remainder, if any, put the next figure of the dividend, and divide this number as before; set down the quotient, and, on the right of the remainder, put the next figure of the dividend ; and so on till the whole division is completed.

To prove, multiply the quotient by the divisor, and add to the product the final remainder, if any: and the result ought to be the dividend.

Ex. (1.) Divide 2356892 by 7.

$$\begin{array}{r} 7\,)\,2356892 \\ \hline 336698 - 6 \\ 7 \\ \hline \text{Proof}\quad 2356892 \end{array}$$

Here, 7 being greater than 2, we say, 7 in 23; the quotient is 3, and remainder 2. Write 3 under the second figure of the dividend. Now 5, placed after the remainder 2, is 25; say, 7 in 25; gives quotient 3, and remainder 4. Write 3 in the quotient, and say, 7 in 46 ; gives quotient 6, and remainder 4. Write down 6 in the quotient. Then 7 in 48, gives quotient 6, and remainder 6. Write down 6. And then 7 in 69, gives quotient 9, and remainder 6. Write down 9. Then 7 in 62, leaves

remainder 6, and quotient 8. Write down 8, and put the remainder, 6, outside on the right of the quotient.

In the proof, when we multiply by 7, 6 is added to the first product; thus, $7 \times 8 = 56$; and 6, added to 56, makes 62.

Ex. (2.) Divide 95768348 by 4.

4) 95768348

23942087

4

Proof 95768348

EXAMPLES.

Divide	Divide
(1.) 578349268 by 2.	(2.) 7986399876 by 2.
(3.) 823573137 by 3.	(4.) 2621134789 by 3.
(5.) 748277624 by 4.	(6.) 7684782356 by 4.
(7.) 478235665 by 5.	(8.) 1090732065 by 5.
(9.) 357316247 by 6.	(10.) 1357953132 by 6.
(11.) 478235659 by 7.	(12.) 5713070049 by 7.
(13.) 466472278 by 8.	(14.) 9669162442 by 8.
(15.) 3521075193 by 9.	(16.) 8011082772 by 9.
(17.) 2357213781 by 11.	(18.) 3211238877 by 11.
(19.) 5701963849 by 12.	(20.) 4411448448 by 12.

For further examples, see the examples in Multiplication, in which the multiplier may be made the divisor, the product the dividend; and then the quotient will be equal to the multiplicand.

XXII. When the divisor is a number greater than 12 (called Long Division),

1. Write down the dividend, and draw a curved line on each side of it; place the divisor on the left-hand.

2. Then, beginning at the left-hand, take as many figures of the dividend as will make a number just greater

than the divisor, and see how many times the divisor is contained in it, and write this quotient on the right-hand of the dividend.

3. Multiply the divisor by this quotient, and place the product under the first figures of the dividend, reckoning from the left-hand; draw a line beneath, and subtract the lower from the upper.

4. Bring down the next figure of the dividend, or more if necessary, to the right of the remainder, and then see how many times the divisor is contained in it, and put the quotient to the right-hand of the former quotient figure.

5. Multiply the divisor by the new quotient, and subtract as before, till every figure of the dividend has been brought down.

The proof is the same as in Short Division.

Ex. Divide 58732 by 87.

87) 58732 (	675	
522	87	
653	4725	
609	5400	
442	7	Remainder
435	58732	
7		

Here, 87 being greater than 58, we take three figures of the dividend, and try how often 87 is contained in 587. The best way to do this, is to find how often 8, the first figure of the divisor, is contained in 58, the first two figures of the dividend. Now, 8 is contained in 58 7 times; but the quotient figure 7, multiplied into 87, will produce a number (609) greater than 587; we therefore take 6 for the first quotient figure; and $87 \times 6 = 522$. Write this under 587; subtract; the remainder is 65. Bring down 3, and try how often 87 is contained in 653; or 8 in 65. Here, although 8 is contained in 65, 8 times, we try 7, and put it in the quotient; and $87 \times 7 = 609$; this subtracted leaves 44. The last

figure, 2, is brought down, and we ask how often is 8 contained in 44; the quotient 5 is written after 7; and 435 subtracted from 442, leaves 7 for the final remainder.

The proof of the correctness is found by multiplying 675 by 87, and adding the remainder 7.

Two cautions are to be given to beginners:—

1st. That the number to be subtracted must always be less than the number above it.

2d. That the remainder, after subtraction, must always be less than the divisor.

XXIII. Long division may be superseded by short division, when the divisor is readily divided into factors.

There is a difficulty in this method, whenever one factor, at least, does not give a complete quotient, since the true remainder cannot always be exhibited by the beginner.

The method may be shown in the following example:—

Divide 7478324 by 36.

Since $36 = 9 \times 4$; so divide by 4, then by 9.

$$\begin{array}{r} 4\,)\,7478324 \\ \hline 9\,)\,1869581 \\ \hline 207731 - 2 \end{array}$$

EXAMPLES.

	Divide		Divide
(21.)	975128578 by 17	(22.)	695093512 by 19
(23.)	192728575 by 23	(24.)	418246803 by 34
(25.)	298357532 by 46	(26.)	725369012 by 39
(27.)	578578196 by 57	(28.)	369258143 by 65
(29.)	240347236 by 71	(30.)	925073243 by 82
(31.)	519390045 by 64	(32.)	242635798 by 43
(33.)	217927618 by 77	(34.)	389659053 by 92
(35.)	347236519 by 96	(36.)	692581439 by 87
(37.)	611635211 by 123	(38.)	3457069256 by 234
(39.)	4570692568 by 542	(40.)	9258107981 by 476

Divide	Divide
(41.) 1832761852 by 379	(42.) 1234567890 by 878
(43.) 7038059076 by 449	(44.) 4564870337 by 279
(45.) 4139368252 by 726	(46.) 6925078143 by 835
(47.) 9272857553 by 2345	(48.) 8965905323 by 5729
(49.) 5198357429 by 7283	(50.)23456874072 by 35792

For other examples refer to those in Multiplication.

XXIV. When the divisor has cyphers on the right-hand, cut off from the right of the dividend as many figures as there are cyphers on the right of the divisor, and divide the remaining figures of the dividend by the remaining figures of the divisor.

Divide 23456832 by 4000.

4,000)23456,832

Quotient 5864, with remainder 832, or as it is written,

$$5864, \tfrac{832}{4000}$$

Divide 4568324 by 7200.

```
72,00) 45683,24 (634 3524/7200
       432
       ----
        248
        216
        ---
         323
         288
         ---
          35
```

Now, after 35 put 24, and the whole remainder is 3524, which is usually expressed as above.

(51.) Divide 245968321 by 7000 and 5000

(52.) „ 478345873 by 2700 and 12300

(53.) „ 123456987 by 83000 and 4400

(54.) A contractor employs 255 men, and at the end of each week pays them 4590 shillings: How much did each man receive?

(55.) If 6,201,856 visitors were at the Great Exhibition from May 1 (Thursday) to October 11 (Saturday), how many on an average visited it each day?

(56.) Divide the difference between seventeen millions one hundred and eighty, and seven millions eight hundred and one, by the difference between three hundred and two thousand and forty-five, and two hundred and three thousand four hundred and fifty.

ARITHMETICAL TABLES.

PENCE TABLE.

d.		*s.*	*d.*	*d.*		*s.*	*d.*
12	are	1	0	84	are	7	0
20	...	1	8	90	...	7	6
24	...	2	0	96	...	8	0
30	...	2	6	100	...	8	4
36	...	3	0	108	...	9	0
40	...	3	4	110	...	9	2
48	...	4	0	120	...	10	0
50	...	4	2	130	...	10	10
60	...	5	0	132	...	11	0
70	...	5	10	140	...	11	8
72	...	6	0	144	...	12	0
80	...	6	8	150	...	12	6

SHILLING TABLE.

s.		£	*s.*	*s.*		£	*s.*
20	are	1	0	90	are	4	10
30	...	1	10	100	...	5	0
40	...	2	0	110	...	5	10
50	...	2	10	120	...	6	0
60	...	3	0	130	...	6	10
70	...	3	10	140	...	7	0
80	...	4	0	150	...	7	10

TABLES OF WEIGHTS AND MEASURES AND MONEY.

TROY WEIGHT.

24 grains	= 1 pennyweight, (dwt.)
20 pennyweights	= 1 ounce, (oz.) = 480 gr.
12 ounces	= 1 pound, (lb.) = 5760 gr.

This weight is used for weighing gold, silver, and jewels, diamonds excepted; its name is said to be derived from Troyes, a town in France.

APOTHECARIES' WEIGHT.

20 grains	=	1 scruple	= 20 gr.
3 scruples	=	1 dram	= 60 gr.
8 drams	=	1 oz.	= 480 gr.
12 oz	=	1 lb.	= 5760 gr.

Observe, the oz. and lb. are the same in troy and apothecaries' weight.

AVOIRDUPOIS WEIGHT.

$27\frac{11}{32}$ grains	= 1 dram.
16 drams	= 1 oz. = $437\frac{1}{2}$ gr.
16 oz.	= 1 lb. = 7000 gr.
28 lbs.	= 1 quarter.
4 quarters	= 1 hundredweight, (cwt.)
20 cwt.	= 1 ton.

This is the weight in ordinary use. Since a lb. troy contains 5760 grains, and a lb. avoirdupois contains 7000 grains; and as these numbers are as 144 to 175; we see the lb. avoirdupois exceeds the lb. troy in the proportion of 175 to 144; in fact, the lb. avoirdupois = 1 lb. 2 oz. 14 dwt. 16 gr. troy.

The carat for weighing diamonds is = $3\frac{1}{6}$ grains.

The legal stone is 14 lbs., but in London, the stone by which meat is sold in Smithfield = 8 lbs. The sack of flour contains 20 stone, or 280 lbs. The following particular weights are used in weighing Wool;

14 pounds	=	1 stone.
2 stone	=	1 tod.
6½ tods	=	1 wey.
2 weys	=	1 sack.
12 sacks	=	1 last.

MONEY.

4 farthings	= 1 penny	=	1*d.*	
12 pence	= 1 shilling	=	1*s.*	= 48*f.*
20 shillings	= 1 pound	=	1*l.*	= 960*f.*

This is the division of money as far as reckoning goes; the coinage is

GOLD.

		dwts.	gr.
Sovereign	weighing	5	$3\frac{1}{4}$
Half-sovereign	. . .	2	$13\frac{1}{2}$

SILVER.

Crown	weighing	18	$4\frac{4}{11}$
Half-crown	. . .	9	$2\frac{2}{11}$
Shilling	. . .	3	$15\frac{3}{11}$
Sixpence	. . .	1	$19\frac{7}{11}$
Fourpence	. . .	1	$5\frac{1}{11}$

The Gold Coinage is said to be 22 carats fine, which means that any gold coin consisting of 24 parts, has 22 parts of fine gold, and 2 parts copper (alloy). The standard used for watches is 18 carats fine; or, an oz. of such gold contains a fourth part alloy. In our coinage 40 lbs. troy of the mixed gold and copper are coined into 1869 sovereigns. In the silver coinage, 11 oz. 2 dwts. of fine silver are melted with an alloy of 18 dwts. of copper: and 1 lb. troy of this compound is coined into 66 shillings. In the copper coinage, 24 pence are coined from an avoirdupois pound of copper. A Guinea, though no longer a legal coin, is reckoned worth 21 shillings.

MEASURE OF LENGTH.

12 inches	=	1 foot.
3 feet	=	1 yard.
$5\frac{1}{2}$ yards	=	1 pole, perch, or rod.
40 poles	=	220 yards = 1 furlong.
8 furlongs	=	1760 yards = 1 mile.
$69\frac{1}{10}$ miles	=	60 geographical miles = 1 degree.
360 degrees	=	circumference of the earth at the equator.

To these may be added,

6 feet, or 2 yards	=	1 fathom.
4 poles, or 22 yards	=	792 inches = 1 chain.
10 chains	=	1 furlong.

Also,

5 feet	=	1 geometrical pace.
4 inches	=	1 hand.

The chain is divided into 100 parts, called links, and therefore each link is $\frac{792}{100}$, or 7·92 inches.

CLOTH MEASURE.

$2\frac{1}{4}$ inches	=	1 nail.
4 nails, or 9 inches	=	1 quarter.
4 quarters	=	1 yard.
3 quarters	=	1 ell, Flemish.
5 quarters	=	1 ell, English.
6 quarters	=	1 ell, French.

MEASURE OF SURFACE, OR SQUARE AND LAND MEASURE.

144 square inches	=	1 square foot.
9 square feet	=	1 square yard.
$30\frac{1}{4}$ square yards	=	1 square pole.
40 square poles	=	1 square rood.
4 square roods	=	1 square acre.
640 square acres	=	1 square mile.

A square has four equal sides, and all its angles right angles; so that a square pole is a square of land $5\frac{1}{2}$ yards every way, and as a chain is 4 poles, a square chain is 16 poles, but a square acre = 160 square poles = 10 square chains. Also, a chain is 100 links; therefore, a square chain is 10,000 square links, and a square acre is 100,000 square links; this fact makes the chain so useful in the measurement of land.

A square mile contains 320 poles every way, and is equal to 320 × 320 square poles = 640 × 160 square poles; but a square acre = 160 square poles, therefore, the square mile = 640 square acres.

SOLID, OR CUBICAL MEASURE.

1728 cubic inches	=	1 cubic foot.
27 cubic feet	=	1 cubic yard.

MEASURES OF CAPACITY.

4 gills	= 1 pint	= $34\frac{2}{3}$	cubic inches, nearly.
2 pints	= 1 quart	= $69\frac{1}{3}$	. . .
4 quarts	= 1 gallon	= $277\frac{1}{4}$	. . .
2 gallons	= 1 peck	= $554\frac{1}{2}$	. . .
4 pecks	= 1 bushel	= $2218\frac{1}{2}$	. . .
8 bushels	= 1 quarter	= $10\frac{1}{4}$	cubic feet, nearly.

To these may be added,

5 quarters	= 1 load
2 loads	= 10 quarters = 1 last.

The first three denominations are used in liquids and in dry goods, the last four only in dry goods. The imperial gallon contains 277·274 cubic inches, being equal in volume to a volume of distilled water which weighs 10 lbs. avoirdupois, at temp. 62° Fahr., the barometer standing at 30 inches; consequently, the imperial bushel contains 80 lbs. of water of the same temperature. In Beer, the following measures are used:—

9 gallons	=	1 firkin.
18 gallons	=	1 kilderkin.
36 gallons	=	1 barrel.
54 gallons	=	1 hogshead.
108 gallons	=	1 butt.

There is no definite number of gallons for the hogshead, pipe, butt, &c., when applied to wine or spirits.

TIME.

60 seconds		= 1 minute.
60 minutes		= 1 hour.
24 hours		= 1 day.
7 days		= 1 week.
365 days	= 52 wks. 1 dy.	= 1 common year.
366 days	= 52 wks. 2 dys.	= 1 leap year.

Leap years are those exactly divisible by 4; they may be found by dividing the *last two* figures of the year by 4; if nothing remains, it is leap year; if there be a remainder, it gives the number of years past leap-year; thus, in the years 1851 and 1852, 51 divided by 4 gives remainder 3; 52 divided by 4 gives no remainder; hence, 1851 is 3 years past a leap-year, and 1852 is a leap-year. The century year is, however, not a leap-year, except the number of the century is divisible by 4; thus, 1900 is not a leap-year, but 2000 is, because 19 is not, but 20 is divisible by 4.

Some of the remarks in these tables can only be understood by the advanced student; they are put here for the purpose of reference; what may be comprehended by the young arithmetician is easily seen.

CHAPTER II.

REDUCTION.

XXV. REDUCTION changes numbers of one or more denominations into a number of one denomination; or, conversely, changes a number of one denomination into a number of one or more denominations.

When a number consists of different denominations, such as, *5l.* 16*s.* 7*d.*, or 10 lbs. 8 oz., it is called a compound number.

There are two cases, 1st, when the number is reduced to one of a lower denomination, in which case we multiply; and 2d, when the number is changed into one of a higher denomination, when division is made use of.

Thus, had we to find how many shillings are contained in a given number of pounds sterling, we must multiply the pounds by 20.

But if we are required to find how many shillings are contained in a given number of pence, we must divide the number of pence by 12.

Ex. (1). How many pence are contained in *25l.*?

We first multiply by 20, and then find the number of shillings in *25l.*, and then multiply this result by 12 to find the number of pence.

£
25
20

500
12

6000

Ex. (2.) Find the number of farthings in 18*l.* 16*s.* $2\frac{3}{4}d.$

£	*s.*	*d.*
18	16	$2\frac{3}{4}$
20		
376		
12		
4514		
4		
18059		

We first multiply 18 by 20, and add 16; which we do in this way: write 20 with the cypher outside; bring down the 6 under the cypher; multiply 18 by 2, and add the 1 of the 16. Again, multiply 376 by 12, adding the 2; multiply 4514 by 4, and add in the $\frac{3}{4}$; then 18059 is the number of farthings contained in 18*l.* 16*s.* $2\frac{3}{4}d.$

Ex. (3.) Reduce 35283 farthings to pounds.

4) 35283

12) 8820 — $\frac{3}{4}$

2,0) 73,5 — 0

£36 15 $0\frac{3}{4}$

First we divide by 4, because 4*f.* make 1*d.*; the quotient is 8820*d.* and the remainder 3 is 3 farthings, or $\frac{3}{4}$; next, we divide 8820 by 12, since 12*d.* make 1*s.*; the quotient is 735*s.*, there is no remainder; lastly, we divide 735 by 20, or by cutting off the cypher and the 5, we divide 73 by 2, the quotient is 36*l.*, and the remainder 1 is 10*s.*, which added to 5, makes 15 for the whole remainder; after dividing 735 by 20, the other remainder is brought down, and the final result is 36*l.* 15*s.* $0\frac{3}{4}d.$

Ex. (4.) In 17 cwt. 2 qrs. 15 lbs. how many lbs.?

cwt.	qrs.	lbs.
17	2	15
4		
70		
28		
565		
141		
1975		

This example needs no explanation, except in the last multiplication, where I add the 5 of the 15 lb. in the units place, when I multiply by 8; and the 1 of the 15 lb. to the first figure of the product, when I multiply by 2.

Ex. (5.) Reduce 123423 feet to miles, furlongs, and yards.

3) 123423

220) 41141 (187 furlongs
220

1914
1760

1541
1540

1 yd.

8) 187

23 — 3

	miles	fur.	yd.
Ans....	23	3	1

We divide by 3, since 3 feet make 1 yard; we then divide the quotient by 220, since 220 yards make a furlong; the quotient is 187 furlongs, and the remainder 1 yard; we then divide 187 by 8, since 8 furlongs

make 1 mile; the quotient is 23, and the remainder is 3, and the whole result is 23 miles, 3 fur. 1 yd.

In practice, the above work may be abbreviated, but abbreviation now would increase the difficulty of the learner.

EXAMPLES.

(1.) In 625*l.* how many shillings?

(2.) In 125*l.* how many pence?

(3.) In 278*l.* how many farthings?

(4.) In 1237*l.* how many sixpences?

(5.) In 2731*l.* how many halfpence?

(6.) In 17*l.* 16*s.* how many shillings?

(7.) In 25*l* 12*s.* 5*d.* how many pence?

(8.) In 31*l.* 17*s.* 6¾*d.* how many farthings?

(9.) In 176 guineas how many shillings?

(10.) In 276*l.* 12*s.* 8¼*d.* how many farthings?

(11.) In 1283*l.* 18*s.* 2¾*d.* how many farthings?

(12.) In 876 guineas how many pence?

(13.) In 1456 guineas how many farthings?

(14.) In 21*l.* 10*s.* 4*d.* how many fourpenny-pieces?

(15.) In 54*l.* 16*s.* 9*d.* how many threepenny-pieces?

(16.) In 85 moidores of 27*s.* each, how many pence?

(17.) In 2783 Spanish dollars at 4*s.* 6*d.* each, how many farthings?

(18.) In 27835 Rupees at 22½*d.* each, how many farthings?

(19.) In 27328 farthings, how many pence?

(20.) In 17864 pence, how many shillings?

(21.) In 278340 shillings, how many pounds?

(22.) In 25683 pence, how many pounds?

(23.) In 52783 farthings, how many pounds?

(24.) In 45 tons 3 cwt. 1 qr., how many pounds?

(25.) In 23 cwt. 2 qrs. 27 lb. how many oz.?

(26.) In 9 cwt. 3 qrs. 14 lb. 7 oz. how many drams?

(27.) How many lbs. of gold are there in 397440 gr.?

(28.) In 17 lb. 5 oz. 3 dwts. 8 gr. troy, how many gr.?

(29.) In 54728 lb. avoirdupois, how many gr ?

(30.) In 3 lb. 7 oz. 6 dr. 2 scrup. 12 gr. (apothecaries' weight,) how many grains.

(31.) In 132480 gr., how many lbs. apothecaries' wt.

(32.) How many weeks are there in 14 common years?

(33.) How many seconds are there in a day, a week, and a year ?

(34.) In 185 days 12 hours and 20 minutes, how many minutes ?

(35.) How many days have elapsed since the creation of the world, up to Dec. 31, 1851, inclusive; our Lord's birth being 4004 A.M. ?

(36.) In 93 ingots of silver, each weighing 2 lb. 9 oz. 17 dwts. how many dwts. ?

(37.) Reduce 72 yds. 2ft. 11 inches to inches.

(38.) Reduce 12 miles 3 furlongs 176 yards to yards.

(39.) Reduce 29 yds. 2 qrs. 3 nails to nails.

(40.) Reduce 20 acres 3 roods 27 poles to poles.

(41.) Reduce 52 nails 7 furlongs 8 chains 2 poles to inches; a pole is 198 inches.

(42.) Find the number of days from Jan. 2, to Oct. 22, in the year 1852.

(43.) In 10 pieces of cloth, each 32 yds. 2 qrs. long, how many inches ?

(44.) In 768532 yards, how many miles ?

(45.) In 27835 qrs. how many yards English, ells Flemish, and ells French ?

(46.) In 18 cub. yds. 19 cub. feet. 29 cub. in. how many cubic inches ?

(47.) Reduce 272 qrs. 7 bus. 3 pks. to pecks.

(48.) In 75634 pints of beer, how many hogsheads, each containing 54 gallons ?

(49.) Reduce 375 hogs. 12 gall. 3 qts. 1 pt. of beer to pints.

(50.) If an individual consume 320 lb. of flour in a year, how many sacks of flour will be consumed by the population of a town containing 25235 inhabitants?

(51.) How many times does a clock strike in a day?

(52.) How many times in a year?

(53.) The population of London, in 1851, being 2,363,141, and one person in 42 dying annually, how many die in a year?

(54.) In the same population, if there be one birth to every 36 inhabitants, how many children are born in a year?

(55.) In the same population, how many die, or are born, weekly, and daily (omitting remainders)?

(56.) How many inches are there in 10 miles?

(57.) The circumference of the earth being 25,000 miles, how many yards are there in the circumference?

(58.) A railway carriage travels at the rate of 60 miles an hour; How many feet does it pass over in a second?

(59.) A person at the equator travels by the earth's rotation 25,000 miles in 23 hours and 56 minutes; What space does he travel through in one second?

(60.) Sound travels at the rate of 1,125 feet in one second; find the distance of the thunder-cloud, when the thunder is heard $8\frac{1}{4}$ seconds after the lightning was visible.

(61.) Light travels at the rate of 192,000 miles in one second; find the time in which light will travel from the sun to the earth, their distance being 96 millions of miles.

(62.) A cannon-ball issues from a gun with a velocity of 2,000 feet in 1″; How long would it be in going to the sun, if it lost none of its velocity?

(63.) An express train travels at the rate of 50 miles an hour; In what time would it travel round the earth's equator?

(64.) And how long would it be in travelling from the earth to the sun?

CHAPTER III.

THE COMPOUND RULES.

COMPOUND ADDITION.

XXVI. Compound Addition teaches us to collect into one sum or amount several numbers of the same kind, consisting of different denominations.

Rule 1.—Write the numbers so that those of the same denomination be in the same column, and draw a line below the last number.

2.—Add together the figures of the lowest denomination; find how many of the next denomination are contained in the sum; write down the remainder, if any, and carry the quotient to the next denomination.

3.—Proceed in the same way with the next column, to which is added the number you carry, and so on till the last column is added up; its sum and the several remainders will be the sum required. Prove as in simple addition.

EXAMPLE.

£	*s.*	*d.*
87	17	$7\frac{3}{4}$
75	18	$7\frac{1}{2}$
25	17	$8\frac{1}{4}$
20	13	$10\frac{1}{2}$
12	5	$7\frac{3}{4}$
£222	13	$5\frac{3}{4}$

We first begin with the farthings, the sum of which is 11, and 11*f.* = $2\frac{3}{4}$*d.*, write down $\frac{3}{4}$ and carry 2*d.* to the pence line; this sum is 41, and 41*d.* = 3*s.* 5*d.*, write down 5*d.* and carry 3 to the shilling line; first add the units of the shillings only, this sum increased by 3 is 33, write down 3, and carry the other 3 to the tens line; the sum is 7, divide this by 2, the quotient is 3, and remainder 1; put the 1 before the former remainder 3, and carry 3 to the column under the £, its sum is 222, and the whole sum is 222*l.* 13*s.* $5\frac{3}{4}$*d.*

The method of adding the shilling column is more expeditious than that in which the tens are added at once; it is obvious that the quotient of tens divided by 2 must be twenties, or pounds.

EXAMPLE IN WEIGHTS AND MEASURES.

cwt.	qrs.	lb.	oz.
26	3	27	4
51	2	3	7
16	1	14	2
12	2	11	12
107	2	0	9

The first column is 25 = 1 lb. 9 oz.; the second column with 1 = 56, or 2 quarters; the third column = 8, and 2 carried, make 10 qrs. = 2 cwt. 2 qrs.; the last column with the 2 carried is 107.

EXAMPLES.

(1.)			(2.)			(3.)		
£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
17	16	$4\frac{1}{4}$	11	5	1	44	15	8
28	4	$11\frac{1}{2}$	25	14	$8\frac{3}{4}$	71	8	$7\frac{1}{2}$
8	12	4	15	16	11	13	12	$6\frac{1}{4}$
19	17	$9\frac{1}{4}$	73	4	$8\frac{1}{2}$	47	5	9
23	5	$7\frac{3}{4}$	12	17	$6\frac{1}{4}$	32	12	$2\frac{1}{2}$
42	16	$10\frac{1}{4}$	5	13	2	84	16	7

(4.)

£	s.	d.
71	17	$0\frac{1}{4}$
44	15	$3\frac{1}{4}$
64	7	$10\frac{1}{2}$
92	5	$6\frac{1}{4}$
11	16	$4\frac{3}{4}$
57	13	$8\frac{3}{4}$

(5.)

£	s.	d.
193	7	10
51	16	$8\frac{3}{4}$
21	13	6
91	15	$4\frac{1}{2}$
210	16	$0\frac{1}{4}$
176	11	$5\frac{3}{4}$

(6.)

£	s.	d.
63	18	$9\frac{3}{4}$
3	16	$11\frac{1}{4}$
798	13	$5\frac{1}{2}$
10	7	$11\frac{1}{2}$
71	5	$3\frac{3}{4}$
84	13	5

(7.)

£	s.	d.
75	18	$7\frac{1}{2}$
101	1	9
31	12	$7\frac{1}{2}$
14	5	$6\frac{1}{2}$
53	16	$10\frac{1}{2}$
70	0	$1\frac{1}{4}$
512	19	0
44	16	$11\frac{3}{4}$

(8.)

£	s.	d.
17	2	$4\frac{1}{4}$
19	4	$7\frac{1}{2}$
13	12	$6\frac{3}{4}$
25	4	11
29	16	1
47	11	4
59	7	$10\frac{1}{2}$
38	17	5

(9.)

£	s.	d.
23	12	1
31	14	$7\frac{1}{2}$
37	6	2
49	18	8
26	4	$2\frac{3}{4}$
47	18	$8\frac{1}{4}$
69	12	7
94	13	$4\frac{1}{2}$

(10.)

£	s.	d.
223	4	11
456	18	$2\frac{1}{4}$
678	4	$10\frac{1}{2}$
359	17	8
53	7	$10\frac{1}{4}$
117	5	$6\frac{1}{2}$
289	19	$7\frac{3}{4}$
857	11	8
400	0	$9\frac{1}{4}$
109	7	11

(11.)

£	s.	d.
356	5	$7\frac{3}{4}$
922	7	$4\frac{1}{2}$
289	7	11
304	14	$6\frac{1}{2}$
202	3	6
294	13	8
147	6	11
263	13	$11\frac{1}{4}$
31	12	$0\frac{3}{4}$
19	5	$7\frac{1}{2}$

(12.)

£	s.	d.
6	11	$9\frac{1}{4}$
3	2	4
75	2	$1\frac{3}{4}$
22	10	$0\frac{1}{2}$
127	9	$10\frac{1}{2}$
712	11	$3\frac{1}{2}$
390	17	$2\frac{1}{4}$
7	1	2
104	6	4
625	3	$5\frac{1}{4}$

(13.)

cwt.	qrs.	lb.
12	2	13
23	1	15
4	0	7
16	2	23
51	1	18
9	3	27

(14.)

lb.	oz.	dr.
11	11	7
12	13	14
8	9	12
27	6	3
71	15	9
12	11	8

(15.)

tons.	cwt.	qrs.
841	15	2
345	6	1
743	12	2
211	1	1
677	19	3
512	11	5

(16.)

lb.	oz.	dwt.	gr.
12	11	19	23
9	7	8	13
7	10	17	19
18	4	16	22
4	3	9	7
20	2	15	17

(17.)

cwt.	qr.	lb.
47	1	27
31	2	13
15	3	9
12	2	18
16	3	25
32	2	7

(18.)

ac.	ro.	po.
67	3	29
27	2	19
39	2	25
37	3	17
25	2	39
96	1	27

(19.)

mil.	fur.	yds.
27	3	198
36	6	212
23	7	115
41	2	39
77	3	219
14	2	12

(20.)

yds.	ft.	inc.
12	2	11
13	1	8
4	2	9
72	1	10
123	2	5
456	1	7

(21.)

hrs.	min.	sec.
17	54	53
27	25	52
81	9	12
176	3	59
12	45	36
18	19	27

(22.)

lb.	oz.	dr.	sc.
14	8	7	2
17	10	6	1
63	5	7	2
15	9	4	1
9	3	5	2
4	0	1	1

(23.)

oz.	dr.	sc.	gr.
11	2	1	19
7	2	2	22
14	6	1	17
13	5	2	8
22	7	1	23
12	3	1	13

(24.)

E. ells.	gr.	nails.
95	4	3
27	2	1
14	3	2
45	4	1
29	1	3
77	4	1

(25.)

yds.	qrs.	nails.
85	2	2
58	1	1
47	2	3
23	3	2
94	0	1
86	1	2

(26.)

qrs.	bsh.	pks.
31	7	3
42	5	1
87	3	2
92	6	1
47	4	2
23	5	2

(27.)

wks.	da.	hrs.	min.
59	4	20	50
27	3	18	47
18	1	12	29
93	6	19	37
47	2	23	52
19	4	17	29

(28.)

gall.	qts.	p:s.
23	3	1
47	2	0
63	3	1
44	2	0
85	1	1
72	1	1

(29.)

sol. yds.	ft.	in.
34	19	274
83	22	325
40	10	518
23	21	1136
77	26	149
80	15	894

(30.)

leag.	mil.	fur.
83	2	7
49	1	6
43	2	5
25	1	3
72	2	6
89	1	7

COMPOUND SUBTRACTION.

XXVII. Compound Subtraction teaches us to find the difference between two numbers of the same kind, consisting of different denominations.

Rule 1.—Write down the greater number, and under it the less, so that the numbers of the same denomination stand directly under each other; and draw a line below them.

2.—Begin at the right-hand, and subtract each number in the lower line from the number above it, and set down the remainders.

3.—But if any number in the lower line be greater than the number above it, add to the upper number as many as make one of the next higher denomination; then perform the subtraction, and set down the remainder.

4.—Then carry a unit to the next number in the lower line, which subtract from the number above it; and so proceed through all the subtractions.

We carry the unit, because it has been borrowed in the previous subtraction.

The proof is the same as in Simple Subtraction.

Ex. Find the difference between 1791*l.* 12*s.* 7*d.* and 834*l.* 9*s.* 5*d.*; also between 272*l.* 4*s.* $2\frac{1}{2}$*d.* and 181*l.* 5*s.* $7\frac{3}{4}$*d.*

(1.)

£	*s.*	*d.*
1791	12	7
834	9	5
£ 957	3	2

(2.)

£.	*s.*	*d.*
272	4	$2\frac{1}{2}$
181	5	$7\frac{3}{4}$
£ 90	18	$6\frac{3}{4}$

In Ex. 1, each number in the lower line is less than each corresponding number in the upper; and the difference is found as in Simple Subtraction. In Ex. 2, we cannot take 3 farthings from 2 farthings; we borrow 1*d.* or 4 farthings, and add 4 to 2 = 6 farthings, and take 3, and the remainder is 3 farthings; write $\frac{3}{4}$*d.* down, and carry 1. 1 and 7 make 8; 8 from 2, you cannot; add 12 to 2, since 12*d.* are 1*s.*; 12 and 2 make 14; 8 from 14 leaves 6; write 6 down, and add 1 to 5. Take 6 from 4 in the shilling column; this you cannot do: borrow 1*l.* or 20*s.*; 20 added to 4 makes 24; 6 from 24 leaves 18; set down 18. Add 1 to the pounds column; it makes 182, which taken from 272, as in Simple Subtraction, leaves remainder 90; and the whole result is 90*l.* 18*s.* $6\frac{3}{4}$*d.*

EXAMPLES.

(1.)

	£	*s.*	*d.*
From	25	7	10
Take	18	4	5

(2.)

	£	*s.*	*d.*
	23	12	4
	14	9	8

(3.)

	£	*s.*	*d.*
	67	4	8
	31	6	9

(4.)

£	s.	d.
19	8	$6\frac{1}{2}$
14	7	$5\frac{1}{4}$

(5.)

£	s.	d.
42	8	$2\frac{1}{2}$
25	10	$4\frac{3}{4}$

(6.)

£	s.	d.
29	15	$6\frac{1}{4}$
17	17	$11\frac{3}{4}$

(7.)

£	s.	d.
574	12	$2\frac{1}{2}$
325	4	$9\frac{1}{4}$

(8.)

£	s.	d.
273	8	$4\frac{1}{4}$
196	15	$8\frac{1}{2}$

(9.)

£	s.	d.
1234	14	$3\frac{1}{4}$
969	18	$8\frac{1}{2}$

(10.)

lbs.	oz.	dwt.	gr.
27	7	18	12
19	9	19	5

(11.)

cwt.	qrs.	lb.
25	2	5
16	3	26

(12.)

lb.	oz.	dr.
212	10	11
197	11	13

(13.)

oz.	dr.	sc.	gr.
15	6	1	18
7	7	2	23

(14.)

lb.	oz.	dr.	sc.
25	10	3	1
15	11	6	2

(15.)

mil.	fur.	po.
273	4	23
185	6	39

(16.)

yds.	qrs.	nls.
453	2	1
267	3	2

(17.)

ac.	ro.	po.
629	2	27
437	2	32

(18.)

qrs.	bush.	pks.
4523	2	2
2789	6	3

(19.)

da.	hrs.	min.
364	19	12
265	17	30

(20.)

hrs.	min.	sec.
471	12	35
273	21	54

(21.)

sol. yds.	ft.	in.
2581	12	982
1695	25	1081

COMPOUND MULTIPLICATION.

XXVIII. Compound Multiplication is the method of finding the amount of a compound number, taken a given number of times.

1st, When the multiplier does not exceed 12:

Rule 1.—Put the multiplier under the lowest denomination of the multiplicand, and draw a line under them.

2.—Multiply the number in this denomination by the multiplier; change the result into the next higher denomination, and write down the remainder, if any.

3.—Multiply the next higher denomination by the multiplier, and carry to the product the whole number of the former product, and proceed with the sum as before; continue the process until the highest denomination has been multiplied by the multiplier. The last product, with the several remainders, will be the product required.

Ex. Multiply £2 15*s.* 2½*d.* by 7.

£	*s.*	*d.*
2	15	2½
		7
19	6	5½

The work is thus performed: ½*d.* being 2 farthings, say, 7 times 2 are 14, 14 farthings are 3½*d.*; put down ½ and carry 3. 7 times 2 are 14, add 3, 14 and 3 are 17, and 17*d.* are 1*s.* and 5*d.*; put down 5, and carry 1. 7 times 5 are 35, add 1, makes 36; put down 6, and carry 3. 7 times 1 are 7, and 3 make 10; divide 10 by 2, and the quotient is 5, and no remainder; multiply 2 by 7, and add 5, and the sum is 19.

EXAMPLES.

	Multiply £	*s.*	*d.*				Multiply £	*s.*	*d.*		
(1.)	47	3	9½	by	2	(2.)	23	13	2	by	3
(3.)	180	16	4¼	by	4	(4.)	12	3	7½	by	5

	Multiply £	s.	d.			Multiply £	s.	d.		
(5.)	82	11	$2\frac{1}{4}$	by	6	(6.) 41	17	$8\frac{1}{2}$	by	7
(7.)	235	12	$6\frac{1}{4}$	by	8	(8.) 473	2	4	by	9
(9.)	31	11	$11\frac{3}{4}$	by	10	(10.) 86	12	$4\frac{1}{4}$	by	11
(11.)	73	18	$9\frac{3}{4}$	by	12					

XXIX. When the multiplier exceeds 12, but is the product of two numbers each less than 12, multiply the given compound number by one of the two factors, and then multiply their product by the remaining factor; the result is the amount required.

Multiply 4*l.* 16*s.* 8*d.* by 35.

Here $35 = 7 \times 5$; therefore 7 and 5 are the two factors.

£	s.	d.
4	16	8
		7
33	16	8
		5
169	3	4

EXAMPLES.

	Multiply £	s.	d.			Multiply £	s.	d.		
(12.)	2	19	4	by	28	(13.) 12	18	2	by	56
(14.)	63	2	$4\frac{1}{2}$	by	63	(15.) 52	3	$4\frac{3}{4}$	by	36
(16.)	41	18	6	by	96	(17.) 7	17	11	by	121
(18.)	242	14	$2\frac{1}{4}$	by	42	(19.) 72	12	$3\frac{1}{2}$	by	120
(20.)	24	12	$9\frac{1}{4}$	by	132	(21.) 13	3	$3\frac{3}{4}$	by	144

XXX. When the number is not composed of two factors, or is not a composite number, and does not exceed 155, *i.e.* $12 \times 12 + 11$; take the next lowest number

which may be so divided, and multiply the given quantity by the two factors, as in the preceding examples, and add to the result the product of the top line, by the excess of the given multiplier over the composite number.

Multiply 12*l.* 10*s.* 8*d.* by 38.

Here $38 = 36 + 2 = 3 \times 12 + 2$.

£	*s.*	*d.*	
12	10	8	× 2
		3	
37	12	0	
		12	
451	4	0	
25	1	4	
476	5	4	

Here, after multiplying successively by 3 and 12, the top line is multiplied by 2, and the product, added to that arising from multiplying by 36, gives the amount required.

EXAMPLES.

	Multiply £	*s.*	*d.*				Multiply £	*s.*	*d.*		
(22.)	9	15	$4\frac{1}{4}$	by	29	(23.)	3	7	$2\frac{1}{2}$	by	23
(24.)	21	19	7	by	86	(25.)	2	15	$6\frac{1}{2}$	by	93
(26.)		12	$7\frac{1}{2}$	by	131	(27.)	1	2	$2\frac{1}{4}$	by	78
(28.)	312	17	$4\frac{1}{2}$	by	152	(29.)	4	7	3	by	117
(30.)	37	12	$11\frac{1}{4}$	by	59	(31.)	2	18	$10\frac{3}{4}$	by	103

XXXI. When the number consists of hundreds, or thousands, &c., find first the amount as if the number only consisted of hundreds or thousands, the remaining parts are found separately; the method is best indicated by an example.

Multiply 4*l.* 16*s.* 9½*d.* by 4567.

£	*s.*	*d.*	
4	16	9½	× 7
		10	
48	7	11	× 6
		10	
483	19	2	× 5
		10	
4839	11	8	
		4	
19358	6	8	= amount of 4000
2419	15	10	= . . . 500
290	7	6	= . . . 60
33	17	6½	= . . . 7
22102	7	6½	

EXAMPLES.

	Multiply £	*s.*	*d.*				Multiply £	*s.*	*d.*		
(32.)	78	14	9¼	by	249	(33.)	37	18	10½	by	265
(34.)	7	13	7½	by	475	(35.)	16	3	8¾	by	703
(36.)	12	15	4¼	by	1327	(37.)	1	5	0½	by	2346
(38.)	4	17	3½	by	7689	(39.)	3	15	11¼	by	9823

XXXII. The preceding examples have been of money, the same method applies when the compound quantity is in terms of other things, as weight, capacity, and length.

(40.)	7 cwt. 2 qrs. 17 lb.	by	2, 7, 9, 12
(41.)	4 mil. 2 fur. 87 yds.	by	3, 5, 8, 11
(42.)	17 cwt. 1 qr. 15 lb.	by	83
(43.)	2 ton. 12 cwt. 3 qrs. 18 lb.	by	217
(44.)	7 oz. 4 dwt. 19 gr.	by	171
(45.)	1 dr. 2 scr. 13 gr.	by	73

	Multiply			
(46.)	11 ac.	2 ro.	31 po.	by 27
(47.)	27 yds.	3 qrs.	2 na.	by 325
(48.)	120 qrs.	7 bus.	3pks.	by 2135
(49.)	18 hrs.	25 min.	42 sec.	by 52
(50.)	52 dys.	12 hrs.	17 min.	by 87

COMPOUND DIVISION.

XXXIII. Compound Division teaches us to find how many times a given number is contained in a compound quantity.

Rule 1.—Arrange the divisor and dividend as in Simple Division.

2.—Begin at the left hand; find how many times the divisor is contained in the highest denomination of the dividend, and write down the quotient.

3.—Reduce the remainder, if any, to the next denomination, and add the result to the number which is in that denomination.

4.—Divide this number by the divisor, and write down the quotient, and treat the remainder as before; and so continue the process to the lowest denomination, when the whole quotient will be the sum of the separate quotients.

As in Multiplication, there are three cases; first, when the divisor does not exceed 12; second, when it exceeds 12, but is the product of two numbers, each of which does not exceed 12; and third, when the number is not the product of two such numbers, or exceeds 144, or is not a multiple of 10.

Divide 41*l.* 2*s.* 9*d.* by 9.

	£	*s.*	*d.*
9)	41	2	9
£	4	11	5

Here 9 is contained in 41 four times, with remainder 5; 5*l.* = 100*s.*; to this add 2, and divide 102 by 9; the quotient is 11, with remainder 3; 3*s.* = 36*d.*; add 9, the sum is 45, which divided by 9, gives a quotient 5, without a remainder.

EXAMPLES.

	Divide						Divide					
	£	*s.*	*d.*				£	*s.*	*d.*			
(1.)	217	12	$8\frac{1}{2}$	by	2	.	(2.) 245	5	3	by	2	
(3.)	36	16	$4\frac{1}{2}$	by	3		(4.) 175	9	$5\frac{1}{4}$	by	3	
(5.)	251	18	8	by	4		(6.) 39	7	11	by	4	
(7.)	119	14	$3\frac{1}{4}$	by	5		(8.) 131	12	11	by	5	
(9.)	162	19	9	by	6		(10.) 5	8	$4\frac{1}{2}$	by	6	
(11.)	331	13	1	by	7		(12.) 663	6	$7\frac{1}{4}$	by	7	
(13.)	430	15	8	by	8		(14.) 193	3	0	by	8	
(15.)	331	7	$4\frac{1}{2}$	by	9		(16.) 679	6	$5\frac{1}{4}$	by	9	
(17.)	1230	9	7	by	10		(18.) 615	4	$9\frac{1}{2}$	by	10	
(19.)	150	11	3	by	11		(20.) 838	11	$6\frac{3}{4}$	by	11	
(21.)	325	19	6	by	12		(22.) 53	16	9	by	12	

XXXIV. *Case* 2.—When the divisor is the product of two factors, each less than 12, divide the given quantity by one factor, and that quotient by the remaining factor; the second quotient is the one required.

The proof of this, as of the preceding rule, is by multiplying the last quotient by the divisor.

Divide £135 17*s.* $7\frac{1}{4}$*d.* by 35.

$$35 = 5 \times 7$$

	£	*s.*	*d.*
7)	135	17	$7\frac{1}{4}$
5)	19	8	$2\frac{3}{4}$
£	3	17	$7\frac{3}{4}$

EXAMPLES.

	Divide £	s.	d.				Divide £	s.	d.		
(23.)	107	17	$9\frac{3}{4}$	by	15	(24.)	315	16	$2\frac{1}{4}$	by	21
(25.)	770	13	$3\frac{1}{2}$	by	22	(26.)	255	0	$2\frac{1}{4}$	by	27
(27.)	36	7	$8\frac{1}{2}$	by	35	(28.)	136	8	$8\frac{1}{4}$	by	77
(29.)	156	9	0	by	63	(30.)	2264	5	$11\frac{1}{4}$	by	45
(31.)	5239	0	3	by	54	(32.)	208	0	0	by	96
(33.)	1449	9	0	by	108	(34.)	1074	18	0	by	144

XXXV. *Case* 3.—When the number cannot be divided into factors, we proceed as in the annexed example.

Divide £375 10*s.* $5\frac{1}{2}$*d.* by 46.

	£	s.	d.		£	s.	d.
46)	375	10	$5\frac{1}{2}$	(	8	3	$3\frac{1}{4}$
	368						
	7						
	20						
	150	(3*s.*					
	138						
	12						
	12						
	149	(3*d.*					
	138						
	11						
	4						
	46	(1*f.*					
	46						

Here 375, divided by 46, gives a quotient 8*l.* mainder 7; 7*l.* reduced to shillings = 140; add 10 from the dividend, and divide their sum, or 150, by 46, the quotient is 3, and remainder 12; 12*s.* = 144*d.*; add 5 from the dividend, the sum is 149; divide 149 by 46, the quotient is 3, and remainder 11; 11*d.* = 44*f.*, add 2 from the dividend, and the sum is 46; 46 divided by 46 gives a quotient 1*f.* and no remainder.

The total quotient is 8*l.* 3*s.* $3\frac{1}{4}$*d.*]

EXAMPLES.

	Divide £	s.	d.				Divide £	s.	d.		
(35.)	92	6	$8\frac{1}{2}$	by	23	(36.)	185	11	$8\frac{3}{4}$	by	19
(37.)	157	13	$10\frac{1}{4}$	by	17	(38.)	53	16	$6\frac{3}{4}$	by	39
(39.)	301	2	$8\frac{1}{4}$	by	43	(40.)	11	6	$1\frac{3}{4}$	by	65
(41.)	45	6	$10\frac{1}{4}$	by	79	(42.)	481	3	3	by	117
(43.)	751	0	11	by	92	(44.)	2673	1	6	by	348
(45.)	1447	17	6	by	468	(46.)	22102	7	$6\frac{1}{2}$	by	4567
(47.)	3182	5	$3\frac{3}{4}$	by	525	(48.)	8866	0	$0\frac{1}{2}$	by	7081

EXAMPLES OF WEIGHTS AND MEASURES.

					Divide	
(49.)	29 ton.	10 cwt.	102 lb.	2 oz.	by	83
(50.)	835 mil.	1093 yds.	9 in.		by	13
(51.)	53 ton.	1 cwt.	2 qrs.		by	28
(52.)	51 ac.	2 ro.	3 po.		by	51
(53.)	571 yds.	2 qr.	1 na.		by	47
(54.)	290 mo.	0 wks.	0 dys.	10 hrs. 30 min.	by	115

XXXVI. When the divisor is a compound number, reduce both dividend and divisor to the lowest denomination in either, and then divide as in Simple Long Division.

Ex. Divide 401*l.* 4*s.* 3*d.* by 2*l.* 11*s.* $5\frac{1}{4}$*d.*

£	s.	d.	£	s.	d.
2	11	$5\frac{1}{4}$	401	4	3
20			20		
51			8024		
12			12		
617			96291		
4			4		
2469			) 385164 (156		
			2469		
			13826		
			12345		
			14814		
			14814		

Answer, 156 times.

Proof. Multiply £2 11*s.* 5¼*d.* by 156, when the product ought to be 401*l.* 4*s.* 3*d.* Or divide 401*l.* 4*s.* 3*d.* by 156, when the quotient ought to be 2*l.* 11*s.* 5¼*d.*

EXAMPLES.

Divide

	£	*s.*	*d.*		£	*s.*	*d.*
(55.)	27	1	4	by		19	4
(56.)	118	13	0	by	2	16	6
(57.)	96	15	2½	by	2	15	3½
(58.)	481	3	3	by	4	2	3
(59.)	109	16	1¾	by	3	15	8¾
(60.)	177	8	8½	by	7	14	3½
(61.)	920	3	0¼	by	29	13	7¾
(62.)	105	11	9¾	by	1	13	6¼

(63.) A man spends 669*l.* 3*s.* 4*d.* in a year; how much is that a day?

(64.) I spent 1*l.* 13*s.* 3½*d.* in sugar, each lb. costing 8½*d.*; how many lb. did I buy?

(65.) In 97 waggons 2302 cwt. 3 qrs. 15 lb. were carried; what did each waggon carry?

(66.) If 35 casks of sugar cost 80*l.* 8*s.* 6½*d.*, what did each cost?

(67.) If 120 oz. of silver cost 31*l.* 17*s.* 6*d.*, what is that per oz.?

(68.) How many yards at 1*s.* 10¼*d.* a yard may be bought for 7*l.* 13*s.* 10¾*d.*?

CHAPTER IV.

PRACTICE.

XXXVII. PRACTICE is a brief method of answering questions in arithmetic when the value of a unit is given, which are otherwise worked out by tedious multiplications.

It is usually performed by breaking up the price or quantity of an article, into parts, which are divisors of some unit of price or quantity.

These parts are called *aliquot* parts; aliquot signifying the how many times.

Thus, 6*d.* being contained twice in a shilling, is called an aliquot part of a shilling; and 14 lb. being the half of a quarter, or the eighth part of a cwt., is an aliquot part of a quarter, or of a cwt.

If the proposed price be an aliquot part of a pound or shilling, or if the quantity be an aliquot part of the unit of quantity, no reduction into parts is necessary.

The following are the aliquot parts of a pound, a shilling, and a penny, commonly used; the third table contains the aliquot parts of a cwt.

10*s.* 0*d.* is $\frac{1}{2}$ of 1*l.*	6*d.* is $\frac{1}{2}$ of 1*s.*	2 qrs. are $\frac{1}{2}$ of a cwt.
6*s.* 8*d.* is $\frac{1}{3}$ of 1*l.*	4*d.* is $\frac{1}{3}$ of 1*s.*	1 qr. is $\frac{1}{4}$ of a cwt.
5*s.* 0*d.* is $\frac{1}{4}$ of 1*l.*	3*d.* is $\frac{1}{4}$ of 1*s.*	14 lb. is $\frac{1}{8}$ of a qr.
4*s.* 0*d.* is $\frac{1}{5}$ of 1*l.*	2*d.* is $\frac{1}{6}$ of 1*s.*	14 lb. is $\frac{1}{8}$ of 2 qrs.
3*s.* 4*d.* is $\frac{1}{6}$ of 1*l.*	1½*d.* is $\frac{1}{8}$ of 1*s.*	14 lb. is $\frac{1}{8}$ of 3 qrs.
2*s.* 6*d.* is $\frac{1}{8}$ of 1*l.*	1*d.* is $\frac{1}{12}$ of 1*s.*	14 lb. is $\frac{1}{8}$ of 1 cwt.
2*s.* 0*d.* is $\frac{1}{10}$ of 1*l.*	2*f.* is $\frac{1}{8}$ of 1*d.*	16 lb. is $\frac{1}{7}$ of a cwt.
1*s.* 8*d.* is $\frac{1}{12}$ of 1*l.*	1*f.* is $\frac{1}{4}$ of 1*d.*	8 lb. is $\frac{1}{14}$ of a cwt.

A simple example will illustrate our meaning of breaking up the price into aliquot parts. Suppose we wished to find the value of 100 lb. at 9*d.* per lb.

Now 9*d.* is the same as 6*d.* and 3*d.*; and the value would be the same, if, instead of each article costing 9*d.*, 2 articles were bought at 6*d.* and 3*d.* each; but 6*d.* is the half of a shilling; and 100 things at 1*s.* each, will be 100*s.*; and therefore, at 6*d.* each, will amount to half that sum, or 50*s.*; and 100 things at 3*d.* will amount to only half as much as 100 things at 6*d.*, or to 25*s.*; so that 100 things at 9*d.* will cost as much as 100 at 6*d.* and 100 at 3*d.*, or will amount to 50*s.* and 25*s.*, or 75*s.*

Ex. (1.) The work is performed concisely as below.

6*d.*	½	100 at 9*d.* each.	Write down the 100, draw a line under it.
3*d.*	½	50	Take 6*d.* = ½ of a shilling.
		25	3*d.* = ½ of 6*d.*
	2,0)	7,5	The sum of their results will give the value of 100 things at 9*d.*
		£3 15 0	

The top line represents 100 at 1*s.* each; hence, we take 6*d.* as ½; since, as above, 100 at 6*d.* must be half the value of 100 at 1*s.*

(2.) Find the value of 3268 at 10½*d.* each.

6*d.*	½	3268 at 10½*d.* each.	Here 3268 in the top line means 3268 at 1*s.* each.
3*d.*	½	1634	Draw a line.
1½*d.*	½	817	Then 6*d.* = ½ of 1*s.*
		408 6	3*d.* = ½ of 6*d.*
	2,0)	285,9 6	1½*d.* = ½ of 3*d.*
		£142 19 6	and 6*d.* + 3*d.* + 1½*d.* = 10½*d.*

In the third line of this example, the remainder being 1*s.*, is reduced to pence; and the division by 20 of the sum in both examples is necessary to bring the result to pounds, as the unit is a shilling.

(3.) Find the value of 2756 at $\frac{1}{4}d.$ each. A farthing being the fourth of $1d.$, $1d.$ is the unit.

$1f.$	$\frac{1}{4}$	2756 at $\frac{1}{4}d.$ each.
		12) 689
		2,0) 5,7 5
		£2 17 5

or 2756 at $\frac{1}{4}d.$ each is 689 at $1d.$ each; and 689 pence are $57s.$ and $5d.$, or $2l.$ $17s.$ $5d.$

(4.) Find the value of 4778 at $15s.$ each. The unit in this case is $1l.$

$10s.$	$\frac{1}{2}$	4778 at $15s.$ each.
$5s.$	$\frac{1}{2}$	2389
		1194 10
		£3583 10

The top line is 4778, at $1l.$ each; a line being drawn under, we find 4778 at $10s.$ each, and 4778 at $5s.$ each, their sum is the answer.

The unit, being pounds, needs no further reduction.

(5.) Find the value of 6834 at $2s.$ $8d.$ each.

$2s.$	$\frac{1}{10}$	6834 at $2s.$ $8d.$ each.
$8d.$	$\frac{1}{3}$	683 8
		227 16
		£911 4

Here $2s.$ is $\frac{1}{10}$ of $1l.$; and $8d.$ is $\frac{1}{3}$ of $2s.$

The unit being $1l.$ the answer is in pounds.

(6.) Find the value of 2756 at $2l.$ $12s.$ $6d.$ each.

$10s.$	$\frac{1}{2}$	2756 at $2l.$ $12s.$ $6d.$ each.
		2
		5512
$2s.$ $6d.$	$\frac{1}{4}$	1378
		344 10
		£7234 10

By multiplying 2756 by 2, we obtain the value of 2756 at $2l.$ each; then 2756 at $12s.$ $6d.$ each, are found as in (4.) and (5.), and their value, added to that of 2756 at $2l.$, gives the value of 2756 at $2l.$ $12s.$ $6d.$ each.

(7.) Find the value of 12 cwt. 3 qrs. 16 lb. at 2*l.* 17*s.* 6*d.* per cwt.

		£	*s.*	*d.*
2 qrs.	$\frac{1}{2}$	2	17	6
				12
		34	10	0
1 qr.	$\frac{1}{2}$	1	8	9
14 lb.	$\frac{1}{2}$		14	$4\frac{1}{2}$
2 lb.	$\frac{1}{7}$		7	$2\frac{1}{4}$
			1	$\frac{1}{4}$
		£37	1	4

First we find the value of 12 cwt. at 2*l.* 17*s.* 6*d.* each; and then we find the value of the 3 qrs. 16 lb., which may be broken up into 2 qrs., 1 qr., 14 lb., and 2 lb.; 2 qr. is $\frac{1}{2}$ cwt., 1 qr. is $\frac{1}{2}$ 2 qr., 14 lb. is $\frac{1}{2}$ 1 qr., and 2 lb. is $\frac{1}{7}$ of 14 lb.. we might have taken the 16 lb. at once, by dividing the top line by 7, since 16 lb. is $\frac{1}{7}$ of a cwt; in this case the work would stand thus.

		£	*s.*	*d.*
2 qrs.	$\frac{1}{2}$	2	17	6
				12
		34	10	0
1 qr.	$\frac{1}{2}$	1	8	9
16 lb.	$\frac{1}{7}$ cwt.		14	$4\frac{1}{2}$
			8	$2\frac{1}{2}$
		£37	1	4

(8.) Find $3756\frac{1}{2}$ at 5*s.* 6*d.* each.

5*s.*	$\frac{1}{4}$	3756 at 5*s.* 6*d.* each.			
6*d.*	$\frac{1}{10}$	939			
		93	18		
			2	9	for $\frac{1}{2}$
		£1033	0	9	

To find the $\frac{1}{2}$, we divide 5*s.* 6*d.* by 2 as below.

	s.	*d.*
2 |	5	6
	2	9

With these examples carefully worked over, the following will present no difficulties.

XXXVIII. When the price is an aliquot part of a shilling and a penny.

			each *d.*				each *d.*
(1.)	5732	at	6	(2.)	2735	at	6
(3.)	8643	at	4	(4.)	3468	at	4
(5.)	7253	at	3	(6.)	7856	at	3
(7.)	9253	at	2	(8.)	6597	at	2
(9.)	5934	at	$1\frac{1}{2}$	(10.)	8239	at	$1\frac{1}{2}$
(11.)	47683	at	1	(12.)	13579	at	1
(13.)	5289	at	$\frac{1}{2}$	(14.)	5783	at	$\frac{1}{4}$
(15.)	12834	at	$\frac{1}{2}$	(16.)	29832	at	$\frac{3}{4}$
(17.)	7856	at	$\frac{3}{4}$	(18.)	68325	at	$\frac{1}{4}$

XXXIX. When the price is not an aliquot part of a shilling.

			each *d.*				each *d.*
(19.)	4685	at	$1\frac{1}{4}$	(20.)	7382	at	$1\frac{5}{4}$
(21.)	2753	at	$1\frac{3}{4}$	(22.)	8543	at	$2\frac{1}{4}$
(23.)	4768	at	$2\frac{1}{2}$	(24.)	5894	at	$2\frac{3}{4}$
(25.)	1359	at	$3\frac{1}{4}$	(26.)	3158	at	$3\frac{1}{2}$
(27.)	4763	at	$3\frac{3}{4}$	(28.)	7259	at	$4\frac{1}{4}$
(29.)	2795	at	$4\frac{1}{2}$	(30.)	3957	at	$4\frac{3}{4}$
(31.)	5834	at	5	(32.)	8542	at	5
(33.)	2573	at	$5\frac{1}{4}$	(34.)	7539	at	$5\frac{1}{2}$
(35.)	9334	at	$5\frac{3}{4}$	(36.)	4455	at	$6\frac{1}{4}$
(37.)	5775	at	$6\frac{1}{2}$	(38.)	7887	at	$6\frac{3}{4}$
(39.)	3539	at	7	(40.)	7783	at	7
(41.)	8224	at	$7\frac{1}{4}$	(42.)	4664	at	$7\frac{1}{2}$
(43.)	7534	at	$7\frac{3}{4}$	(44.)	5999	at	8
(45.)	4669	at	$8\frac{1}{4}$	(46.)	1224	at	$8\frac{1}{2}$
(47.)	6886	at	$8\frac{3}{4}$	(48.)	4782	at	9
(49.)	8432	at	10	(50.)	2579	at	11
(51.)	9752	at	$10\frac{1}{2}$	(52.)	3593	at	$9\frac{1}{4}$

			each d.				each d.
(53.)	5799	at	$10\frac{3}{4}$	(54.)	5683	at	$9\frac{1}{2}$
(55.)	4999	at	$9\frac{3}{4}$	(56.)	25736	at	$10\frac{1}{4}$
(57.)	47638	at	$10\frac{1}{2}$	(58.)	7888	at	$11\frac{1}{4}$
(59.)	5663	at	$11\frac{1}{2}$	(60.)	8577	at	$11\frac{3}{4}$
(61.)	7835	at	$11\frac{3}{4}$	(62.)	58324	at	$11\frac{1}{2}$
(63.)	37849	at	$11\frac{1}{4}$	(64.)	68593	at	$7\frac{3}{4}$
(65.)	72349	at	$3\frac{3}{4}$	(66.)	57832	at	$10\frac{3}{4}$
(67.)	$6810\frac{1}{2}$	at	$6\frac{1}{2}$	(68.)	$4700\frac{1}{2}$	at	$9\frac{1}{2}$
(69.)	$8562\frac{1}{4}$	at	8	(70.)	$7539\frac{3}{4}$	at	9
(71.)	$6254\frac{3}{4}$	at	$10\frac{1}{2}$	(72.)	$12768\frac{1}{2}$	at	$5\frac{1}{2}$

XL. When the price is an aliquot part of a pound.

			each s. d.				each s. d.
(73.)	2763	at	10 0	(74.)	1956	at	10 0
(75.)	6953	at	6 8	(76.)	7982	at	6 8
(77.)	12345	at	6 8	(78.)	5735	at	5 0
(79.)	8648	at	5 0	(80.)	2562	at	4 0
(81.)	1873	at	4 0	(82.)	9235	at	3 4
(83.)	7567	at	3 4	(84.)	3568	at	2 6
(85.)	2572	at	2 6	(86.)	4932	at	2 6
(87.)	5729	at	2 0	(88.)	7654	at	2 0
(89.)	2843	at	1 8	(90.)	3582	at	1 8

XLI. The price shillings, not aliquot parts of a pound.

			each s. d.				each s. d.
(91.)	2537	at	3 0	(92.)	4753	at	3 0
(93.)	8254	at	6 0	(94.)	7755	at	6 0
(95.)	4923	at	7 0	(96.)	9999	at	7 0
(97.)	7333	at	8 0	(98.)	3559	at	8 0
(99.)	4466	at	9 0	(100.)	7299	at	9 0
(101.)	4865	at	11 0	(102.)	2773	at	11 0

			each s.	d.				each s.	d.
(103.)	1242	at	12	0	(104.)	4896	at	12	0
(105.)	1699	at	13	0	(106.)	6565	at	13	0
(107.)	4949	at	14	0	(108.)	9898	at	14	0
(109.)	3030	at	15	0	(110.)	4545	at	15	0
(111.)	3232	at	16	0	(112.)	6465	at	16	0
(113.)	5151	at	17	0	(114.)	9293	at	17	0
(115.)	3663	at	18	0	(116.)	7227	at	18	0
(117.)	5775	at	19	0	(118.)	8338	at	19	0

XLII. When the price is composed of shillings and pence.

			each s.	d.				each s.	d.
(119.)	2754	at	1	1	(120.)	8647	at	1	2
(121.)	1708	at	1	3	(122.)	6433	at	1	4
(123.)	5145	at	1	5	(124.)	1089	at	2	1
(125.)	2892	at	2	2	(126.)	3691	at	2	3
(127.)	3715	at	2	4	(128.)	4977	at	2	5
(129.)	2104	at	3	1	(130.)	2477	at	3	2
(131.)	7563	at	3	3	(132.)	8853	at	3	5
(133.)	3392	at	4	1	(134.)	7753	at	4	2
(135.)	2982	at	4	3	(136.)	5550	at	4	4
(137.)	1233	at	4	5	(138.)	4999	at	2	8
(139.)	6583	at	1	7	(140.)	3865	at	1	6
(141.)	7782	at	2	7	(142.)	2599	at	3	6
(143.)	4770	at	3	7	(144.)	5008	at	4	
(145.)	8530	at	4	7	(146.)	2399	at	1	9
(147.)	4583	at	2	9	(148.)	7700	at	3	9
(149.)	1280	at	2	10	(150.)	8002	at	2	11
(151.)	7631	at	4	11	(152.)	4854	at	4	8
(153.)	9449	at	4	9	(154.)	4118	at	4	10
(155.)	1183	at	5	6	(156.)	6181	at	5	7
(157.)	3009	at	5	11	(158.)	6573	at	3	11
(159.)	7557	at	3	10					

XLIII. The price being pounds, shillings, and pence.

		each. £	s.	d.			each. £	s.	d.
(160.)	2763 at	1	10	8	(161.) 4830 at	2	12	6
(162.)	9763 at	4	6	8	(163.) 2345 at	2	3	4
(164.)	4783 at	4	1	8	(165.) 6660 at	4	7	6
(166.)	9854 at	12	13	4	(167.) 3269 at	2	4	$5\frac{1}{2}$
(168.)	4832 at	2	15	8	(169.) 4786 at	3	10	6
(170.)	7964 at	9	12	$8\frac{1}{2}$	(171.) 2598 at	7	19	$8\frac{1}{2}$
(172.)	7204 at	4	17	$6\frac{3}{4}$	(173.) 4627 at	6	19	8
(174.)	4121 at	1	8	9	(175.) 5437 at	1	6	$1\frac{1}{2}$
(176.)	3478 at	2	5	10	(177.) 2772 at	3	7	$9\frac{1}{2}$
(178.)	1236 at	5	11	$10\frac{1}{2}$	(179.) 1181 at	11	17	$10\frac{3}{4}$
(180.)	$5835\frac{1}{2}$ at	14	12	6	(181.) $2998\frac{3}{4}$ at	15	6	9
(182.)	$4739\frac{1}{4}$ at	2	12	8				

XLIV. When the quantity is a weight or measure.

(183.)	25 cwt.	3 qrs.	14 lb.	at	2	14	8	per cwt.
(184.)	57 cwt.	2 qrs.	16 lb.	at	3	7	6	per cwt.
(185.)	82 cwt.	1 qr.	27 lb.	at	1	14	10	per cwt.
(186.)	18 cwt.	3 qrs.	19 lb.	at	4	12	9	per cwt.
(187.)	19 oz.	10 dwt.	12 gr.	at	3	17	9	per oz.
(188.)	275 oz.	17 dwt.	10 gr.	at	3	17	$10\frac{1}{2}$	per oz.
(189.)	13 oz.	4 dr.	2 sc.	at		5	8	per oz.
(190.)	25 oz.	7 dr.	1 sc.	at		3	6	per oz.
(191.)	57 qrs.	6 bus.	3 pks.	at	2	4	0	per qr.
(192.)	125 qrs.	7 bus.	2 pks.	at	1	17	6	per qr.
(193.)	31 qrs.	4 bus.	1 pk.	at	2	2	5	per qr.
(194.)	57 yds.	3 qrs.	2 na.	at		7	2	per yd.
(195.)	272 yds.	2 qrs.	3 na.	at		1	10	per yd.
(196.)	28 yds.	3 qrs.	3 na.	at	1	12	6	per yd.
(197.)	321 acre.	2 ro.	30 po.	at	42	0	0	per ac.
(198.)	825 ft.	10 in.		at		2	3	a ft.
(199.)	725 ft.	11 in.		at		7	6	a ft.
(200.)	235 ac.	2 ro.	20 po.	at	22	12	6	per ac.

CHAPTER V.

RATIO. PROPORTION. THE RULE OF THREE.

XLV. The Ratio of one quantity to another quantity, is the relation they bear to each other, expressed numerically. To estimate it, we find how many times each quantity contains a given quantity, called a unit. The two numbers so arising form the Ratio of the two quantities. Thus, the Ratio of 8 lb. to 7 lb. is 8 : 7; the common unit being 1 lb.; and the Ratio of 25*l.* to 16*l.* is expressed by 25 to 16; or, 25 : 16, the common unit being here 1*l.*

XLVI. If the two terms of a Ratio are divided by a common divisor, it will not affect the value of the Ratio. Thus the Ratio of 18 lb. to 14 lb., or, 18 : 14 = 9 : 7. For 18 lb. contains 2 lb. 9 times, and 14 lb. contains 2 lb. 7 times; hence, 2 lb. is the common unit, and therefore the ratio of 18 lb. : 14 lb. is as 9 : 7.

The first term of a Ratio is called the antecedent, the second the consequent.

XLVII. Four numbers are said to be in Proportion, when the first contains, or is contained by, the second, as many times as the third contains, or is contained by, the fourth: or when the first is that part of the second that the third is of the fourth.

Thus the three sets of numbers

8,	4,	10,	5.
3,	9,	4,	12.
4,	6,	14,	21.

are in proportion; for in the first set of numbers, the

first and third are respectively double of the second and fourth: in the second set, the first and third are respectively the third parts of the second and fourth; in the third set, the first is two-thirds of the second, and the third is two-thirds of the fourth.

A Proportion is read in this way:—

8 is to 4 as 10 is to 5;

and written thus:—

$$8 : 4 :: 10 : 5.$$

Cor. Since $8 : 4$ is = the Ratio of $2 : 1$
and $10 : 5$ is = the Ratio of $2 : 1$

it appears that when two Ratios are equal, the four terms are in proportion.

Thus if 8 lb. cost 16s. and 7 lb. cost 14s.,

then, since the Ratio of $16 : 14 :: 8 : 7$,

$$\therefore 8\text{ lb.} : 7\text{ lb.} :: 16\text{ s.} : 14\text{ s.}$$

XLVIII. Of the four numbers in a Proportion, the first and fourth are called the extremes, the second and third are called the means; and the *test* of the proportionality of four numbers is that the *product* of the *extremes* is equal to the *product* of the means.

Thus, 8, 4, 10, 5, are in proportion,

and $8 \times 5 = 40 = 4 \times 10$,
also $3 \times 12 = 36 = 9 \times 4$,
and $4 \times 21 = 84 = 6 \times 14$.

Hence, since always in proportion

the first : the second :: the third : the fourth;
and the first × the fourth = the second × the third;

or, dividing both sides of the equality by the first, which obliterates the first from the left-hand member of the equality, we have

$$\text{the fourth} = \frac{\text{the second} \times \text{the third}}{\text{the first}:}$$

or if four terms be in proportion, the fourth is equal to

the product of the second and third divided by the first. This is, in fact, the Rule of Three.

XLIX. As an example. If 27 lb. of tea cost 6*l.* 15*s.* what will be the cost of 43 lb. ?

It is obvious that 27 lb. will bear that ratio to 43 lb. which the cost of 27 lb. will bear to that of 43 lb. ; now the cost of the former is known, being 6*l.* 15*s.* or 135*s.* ; hence we shall have this proportion :—

$$\overset{\text{lb.}}{27} : \overset{\text{lb.}}{43} :: \overset{s.}{135} : \text{cost of } \overset{\text{lb.}}{43}.$$

Therefore, by the rule, the cost of 43 lbs. $= \frac{43 \times 135.}{27}$

We work thus :—

$$\begin{array}{r} 135 \\ 43 \\ \hline 405 \\ 540 \\ \hline 27 \,)\, 5805 \,(\, 215s. = 10l.\ 15s. \\ 54 \\ \hline 40 \\ 27 \\ \hline 135 \\ 135 \\ \hline \end{array}$$

We will now reason out the question by first principles.

Since 27 lb. cost 6*l.* 15*s.* or 135*s.*, 27 lb. must cost twenty-seven times as much as 1 lb. ; or, we must divide the cost of 27 lb. by 27, to obtain the cost of 1 lb.

$$\therefore \text{ the cost of 1 lb.} = \frac{135}{27}.$$

$$\therefore \text{ the cost of 43 lbs.} = 43 \text{ times the cost of 1 lb.}$$

$$= \frac{43 \times 135}{27}$$

the same result as we obtained by the rule.

Now again turn to the example, in which you see that the first and second terms are of the same denomination

both in this example being lb.; the fourth term is of the same denomination as the third, both being money: also, as the price of 43 lb., which is required, is manifestly more than the price of 27 lb., it is put in the second place; had the price of 43 lb. been given, viz. 10*l.* 15*s.*, or 215*s.*, and that of 27 lb. been required, the statement would have been

lb. lb. *s.* lb.
43 : 27 :: 215 : cost of 27.

Here the second term is less than the first, since its value, which is required, is less than the cost of 43 lb.; the proportion would be manifestly wrong, unless the statement was as we have written it.

These remarks being attended to, we shall now give the Rule by which questions of the Rule of Three may be easily solved.

L. *Rule* 1.—Consider which of the three given terms is of the same kind as the answer sought, and make it the third or last term.

2.—Then, as it is easy to see whether the answer be greater or less than the third term, consider whether the answer be greater than the third term; if so, put the greater of the two other given numbers in the second place, and the other first; if, however, the answer be less than the third term, put the less of the two numbers in the second place, and the greater first.

3.—Then, if necessary, reduce the *first* and *second* terms to the *same* denomination; and the third to the lowest denomination in it.

4.—Multiply the second and third terms together, and divide by the first; the quotient will be the answer, or fourth term, but will be expressed in the denomination to which the third term has been reduced.

If necessary, reduce the quotient to a higher denomination.

Ex. (1.) A servant is hired at the rate of 25*l.* 15*s.* for 12 months; what is due to him at the end of 7 months? Here 25*l.* 15*s.* will be the third term: and the wages for 7 months being less than the wages for 12 months, the second term will be 7; and the statement is

months. months. £. s.
12 : 7 :: 25 15
20
515
7
12) 3605
2,0) 30,0 5
£15 0 5

The 5 remainder is 5*s.* or 60*d.*, which, divided by 12, gives 5*d.*

(2.) If a person spend 700*l.* annually, what is his weekly expenditure?

days. days. £
365 : 7 :: 700
7
365) 4900 (13
365
1250
1095
155
20
3100 (8
2920
180
12
2160 (5
1825
335
4
1340 ($\frac{3}{4}$
1095
245

Ans. 13*l.* 8*s.* $5\frac{3}{4}$*d.*

(3.) If 42 men do a certain work in 108 days, how many men will do the same in 63 days?

days. days. men.
63 : 108 :: 42

Since $63 = 9 \times 7$ and $108 = 9 \times 12$

the ratio of $63 : 108 = 7 : 12$

$\therefore 7 : 12 :: 42$

12

7) 504

72 men.

We may always divide the first and second terms by any common divisor: so also the first and third terms.

(4.) A tradesman pays his creditors 4*s.* 3*d.* in the 1*l.*, what will a creditor receive for a debt of 1256*l.* 13*s.* 4*d.*?

£	£	*s.*	*d.*	*s.*	*d.*
1 :	1256	13	4 ::	4	3
20	20			12	
20	25133			51	
3	3				
60	75400				
—	51				
	75400				
	3770				
6,0)	384540,0				
12)	64090				
2,0)	5340	10			
	£267	0	10		

The first and second terms were multiplied by 3, to reduce them to fourpences, instead of reducing them to

pence, which would make both terms larger, but yet in the same proportion.

(5.) Questions of the same kind as (3), in which the number of agents varies inversely as the time, *i.e.*, one increases as the other decreases, possess some difficulty in applying first principles to them.

Ex. If 14 men do a work in 12 days, how many will do it in 7 days?

14 men can do a work in 12 days.

1 man can do $\frac{12}{14}$ part in 12 days.

$\therefore$ $\frac{1}{14 \times 12}$ part in 1 day.

$\therefore$ $\frac{7}{14 \times 12}$ or, $\frac{1}{24}$ in 7 days.

Hence, if one man can do the $\frac{1}{24}$ part in 7 days, 24 men can do the whole in the same time.

EXAMPLES.

(1.) Having bought 19 lb. of tea for 5*l.* 4*s.* 6*d.*, what ought I to pay for 37 lb.?

(2.) If 52 lb. of sugar cost 2*l.* 0*s.* 6*d.*, what is the cost of 10 cwt. 2 qrs. 18 lb.?

(3.) What is the value of a mass of gold weighing 27 lb. 8 oz. 10 dwt. at 3*l.* 17*s.* 9*d.* per oz.?

(4.) A man spends 5*l.* 5*s.* in three days, how much will he have left out of an income of 650*l.* in the year 1852?

(5.) A traveller by railway pays 18*s.* 4*d.* for a journey of 88 miles, what will he have to pay for 243 miles?

(6.) If 15 men could build a wall in 22 days, how many could do the same work in 10 days?

(7.) If the rent of 10 acres 2 roods be 14 guineas, find the rent of a farm of 245 acres 3 poles.

(8.) The income-tax being 7*d.* in the pound, what is the income of a person who pays 18*l.* 19*s.* 2*d.* income-tax?

(9.) If the 4 lb. loaf be sold at 9*d.* when the price of

wheat is 60*s.* a quarter, what ought to be its price when wheat is sold at 40*s.*?

(10.) The price of 1 cwt. of sugar is 2*l.* 6*s.* 8*d.*; find the cost of 47 hogsheads, each weighing 5 cwt. 2 qrs. 17 lb.

(11.) Having bought 5 sacks of flour for 8*l.* 12*s.* 11*d.*, what will be the cost of 117 sacks at the same rate?

(12.) If 13 men could in 10 days reap 65 acres of wheat, how many acres could 42 men reap in the same time?

(13.) The rental of a parish is 25,000*l.*, and a rate produces 2,083*l.* 6*s.* 8*d.*, what is the assessment on a house rated at 125*l.* per annum?

(14.) If 7 yards of cloth cost 3*l.* 8*s.* 1½*d.*, how many yards may be bought for 44*l.* 15*s.* 1*d.*?

(15.) A man's net income is 860*l.* 0*s.* 7½*d.*, what might he spend in a week, if he spend all his income?

(16.) There are provisions for 3 weeks in a fort containing a garrison of 1,000 men; how many ought to be sent away that the provisions may last 5 weeks?

(17.) If a rate of 1*s.* 8*d.* in the 1*l.* be made on a parish, and it produces 1,041*l.* 13*s.* 4*d.*, what is the rental of the parish?

(18.) If 1 ell (English) 2 qrs. of muslin, cost 4*s.* 1*d.*, what will be the cost of 7 pieces, each containing 14½ yards?

(19.) How many yards of paper, 18 inches wide, will paper a room which is four yards high, 6 yards long, and 4 yards wide?

(20.) If 32 bricks will pave a square yard, how many will it require for a court-yard 24 feet wide, and 36 feet long?

(21.) What is a man's net income when his taxes are 2*s.* 9*d.* in the 1*l.*; his gross income being 350*l.* a-year?

(22.) A merchant fails, his debts amount to 56,700; he pays a composition, which amounts to 8,100*l.*; how much will a creditor lose, to whom he owed 2,800*l.* 14*s.* 7*d.*?

(23.) A servant's wages are 30 guineas a-year; what is due to him after 105 days' service?

(24.) A piece of cloth 39 yards long, and 5 quarters wide, is to have a lining 3 quarters wide; how many yards must there be?

(25.) A farm, consisting of 246 acres, 2 roods, is let at 27*s.* 6*d.* per acre; find the half-yearly rent.

(26.) How many yards of carpet, 2 feet 6 inches wide, will be required for a room 30 feet long, and 20 feet wide?

(27.) If 18 men can complete a piece of work in 34 days, in what time would they do it with the assistance of 18 boys, two men being able to do as much work as 3 boys?

(28.) A merchant borrowed 192 quarters of wheat, when the price was 45*s.* 6*d.*; how many must he return when the price was 42*s.*?

(29.) If 7 pipes fill a cistern in 15 hours, in what time will 10 pipes fill it?

(30.) A bankrupt's effects are worth 2,689*l.* 17*s.* 0*d.*, his debts are 9,356*l.*; what will the creditors receive in the pound?

(31.) A person hires lodgings at 17*s.* 6*d.* a-week; what must he pay if he retain them 297 days?

(32.) A bankrupt's debts are 4,786*l.* 13*s.* 4*d.*; what will his creditors receive if the assets produce 7*s.* 6*d.* in the pound?

(33.) Bought 4 cwt. 3 qrs. 14 lb. of sugar, at 55*s.* per cwt.; what is the gain, if it be retailed at 6¼*d.* per lb.?

(34.) If 507 quarters of barley be bought for 701*l.* 7*s.*, how many will 179*l.* 16*s.* 8*d.* purchase?

(35.) If 37 yards of cloth cost 2*l.* 5*s.* 6*d.*, how many yards may be purchased for a guinea and a half?

(36.) The wages of 35 men amount to 21*l.* 17*s.* 6*d.*; find how much the wages of 23 exceeds that of the remainder.

(37.) If 1 cwt. cost 5*l.* 1*s.* 6*d.*, how many casks, each weighing 1 cwt. 2 qrs. 17 lb., may be bought for 67*l.* 1*s.* 3*d.*?

(38.) Bought 33lb. for 8*s.* 11½.; find the cost of 5 cwt. 2 qrs. 18lb.

(39.) A merchant purchased 250 quarters of wheat, at 39*s.* per quarter; how much barley at 27*s.* a quarter must he exchange it for to gain 1*s.* a quarter by his purchase of wheat?

(40.) If 54*l.* 16*s.* 8*d.* be divided among 47 persons, what is the portion which 29 receive?

(41.) If 85 men can do a certain work in 8 days, how many could do it in 7 days.

(42.) If 28 oxen are grazed in a field for 84 days, how many would have been equally well fed for 48 days?

(43.) If 1 oz. of silver cost 5*s.* 6*d.*, find the cost of a tankard weighing 1 lb. 8 oz. 10 dwt. 6 gr.

(44. At 1½ guinea per week, how long could I be boarded for 90*l.*?

(45.) If the penny loaf weigh 8 oz. when wheat is 6*s.* 6*d.* a bushel, what ought it to weigh when wheat is 5*s.* 6*d.* a bushel?

(46.) A garrison of 560 men have provisions for a year; how long will the provisions last when 440 additional men are added?

(47.) The extent of a battalion being 320 feet, and each man marching 75 paces of 2½ feet each per minute, how long would 12 battalions be marching through a defile 2½ miles long?

(48.) If the carriage of 3 cwt. 2 qrs. 14lb. for 40 miles be 18*s.* 5¼*d.*, what will the carriage of 10 tons 3 cwt. be twice the distance?

(49.) If the clothing of 300 men be 1,288*l.* 15*s.*, what will be the cost of clothing 951 men?

(50.) A person buys 256 yards of cloth for 180*l.* and sells it at 18*s.* per yard; what is the gain by the sale?

(51.) A person, whose annual income is 1,000*l.*, spends 1*l.* 11*s.* 6*d.* per day; how much may he lay by?

(52.) If 1 cwt. 2 qrs. 15lb. cost 3*l.* 8*s.* 7½*d.*, find the cost of 3 tons, 14 cwt. 1 qr. 9lb.

(53.) If 6 yards 2 quarters of cloth cost 1*l.* 6*s.* 6½*d.*,

what will 6 pieces, each containing 22 yards 2 quarters, cost?

(54.) If 15 men can do a piece of work in 27 days, how many men can do the same in 35 days?

(55.) If I lend my friend 250*l.* for 5 months, how long ought he, to requite my kindness, to lend me 200*l.*?

(56.) If 10 men, by working 12 hours a-day in summer, can finish a piece of work in $5\frac{1}{2}$ days, in how many days will they do the same work in winter, when they can only work 8 hours a-day?

(57.) If 20 men, by working 11 hours a-day, can finish a piece of work in 6 days, in the summer, how many must be employed in the winter, when they can only work 9 hours a-day, so as to finish the work in the same time?

(58.) If a pipe of port wine, containing 115 gallons, be bought for 90 guineas, what is that per dozen, supposing that 2 gallons are equivalent to $11\frac{1}{2}$ bottles?

(59.) If 6 paces be equal to 5 yards, what distance does a person walk who takes 1,000 paces?

(60.) Three persons travel by railway, one in the first class, paying $2\frac{3}{4}d.$ per mile; another, in the second class, paying only 2*d.* per mile; and the third, in the third class, paying $1\frac{1}{4}d.$ per mile; what was paid by them for a journey of 243 miles?

(61.) A bushel of wheat weighs 63 lb. 4 oz., and is sold for 5*s.* 3*d.*; what is the weight and value of 270 qrs. 5 bushels?

(62.) If 100*l.* gain 4*l.* 10*s.*, what sum of money will gain 16*l.* 10*s.* 9*d.*?

(63.) If 100*l.* gain 3*l.* 15*s.*, what will be the gain on 1,720*l.* 12*s.* 6*d.*?

(64.) If the carriage of 4 cwt. 2 qrs. for 50 miles cost 1*l.* 2*s.* 9*d.*, how far can 14 cwt. 1 qr. be carried for the same money? and then find the cost of its carriage the same distance, namely, 50 miles.

(65.) If 15 yards of cloth cost 7 guineas, how many ells (English) can I buy with 275*l.* 12*s.* 6*d.*

(66.) If 540*l.* 10*s.* produce 27*l.* 0*s.* 6*d.* in 12 months, what sum of money will produce the same in $9\frac{1}{2}$ months?

(67.) If a bullock cost 14*l.* 12*s.* 6*d.*, a sheep 2*l.* 10*s.* 6*d.*, a cow 8*l.* 4*s.* 6*d.*, and a calf 3*l.* 8*s.* 4*d.*, and a man goes to market with 450*l.* and buys an equal number of each, how many does he buy, and what has he over?

(68.) If in Paris the sovereign be worth 25 francs 34 centimes (100 centimes being equal to a franc), how many francs would a merchant receive for 255 sovereigns?

(69.) A tradesman having 10,000 yards of calico, at $4\frac{3}{4}d.$ per yard, wishes to exchange it for silk, at 3*s.* 5*d.* per yard; how many yards of silk ought he to receive?

(70.). Having expended 100*l.* in the purchase of cloth, at 7*s.* 6*d.* per yard, I am compelled to sell it at a loss of 10*l.*; what is the price per yard at which I must sell it?

COMPOUND PROPORTION, OR DOUBLE RULE OF THREE.

LI. In the questions on Simple Proportion it will be observed that the quantity sought depends upon some other single quantity: thus the weight of one article, and its price, being given, either the weight of another article, or its price, is required: here the weight varies as the single quantity the price, or the price varies as the single quantity the weight: so, again, the work done depends upon the number of workmen employed; or it may depend upon the time in which the same body of workmen work: *i. e.* the work may depend either on the time, or the work may depend on the number of agents.

LII. But there are questions in which, to take the last case as an instance, the work done is made to depend, not only on the time, but also on the number of persons employed; or, as it is said, on the time and the number of persons conjointly: this would be a case of Compound Proportion, or of the Double Rule of Three. It will be best to give an example:—

LIII. If 9 men can dig 120 yards in 8 days, how many yards can 24 men dig in 16 days? Here it is obvious that the work depends on the two quantities, the men and the days: and we ought to have two proportions, one involving the work and the men, the other the work and the days. We will first give the Rule, then work the question by it, and afterwards work it independently: and thus justify the assumptions in the Rule.

Rule 1.—Put that number last which is of the same kind as the answer.

2.—For the first and second terms take any two of the terms that are alike, and state as in common Rule of Three; do the same with the other pair, or other pairs of numbers.

3.—Write down as factors all the middle terms, and the last term; and draw a line, and under it put as factors all the numbers in the first terms of the proportion.

4.—Reduce as much as possible by striking out common factors, and then make the top line a dividend, and the lower line a divisor; the quotient will be the answer.

Resuming the question, the statement is as follows:—

$$\begin{array}{ccccc} \text{men} & & \text{men} & & \\ 9 & : & 24 & :: & \text{yds.} \\ \text{days} & & \text{days} & & 120 \\ 8 & : & 16 & :: & \end{array}$$

$$\frac{24 \times 16 \times 120}{8 \times 9} = \frac{3 \times 16 \times 40}{3} = 16 \times 40 = 640. \text{ yds.}$$

Here 120 yards is put last: then we ask, If 9 men can dig 120 yards, how many yards can 24 men dig? This gives the first proportion.

Again, if 120 yards can be dug in 8 days, how many yards can be dug in 16? This will give the second proportion. Then, by the Rule, write down as factors 24, 16, and 120, draw a line, and under it place 9 and 8 as factors:

Then, since $24 = 8 \times 3$; and $120 = 40 \times 3$; $9 = 3 \times 3$; we strike out 8 and 3 from above and below: and afterwards the remaining 3 of the upper and lower line.

LV. We should have obtained the same result had we worked the question in the following manner :—

First, if 9 men can dig 120 yards in 8 days, how many yards can 24 men dig in the same time ?

$$9 : 24 :: 120 : \frac{120 \times 24}{9} \text{ Ans.}$$

Next, if 24 men can dig $\frac{120 \times 24}{9}$ in 8 days, how many yards can they dig in 16 days ? Here

$$8 : 16 :: \frac{120 \times 24}{9} : \frac{120 \times 24 \times 16}{8 \times 9}.$$

The same result as before, and which is of the same form.

If this example be carefully studied, there need be no difficulty in doing or understanding questions in Double Rule of Three.

Ex. (2.) If 252 men can dig a trench 210 yards long, 3 wide, and 2 deep, in 5 days of 11 hours each, in how many days of 9 hours each will 22 men dig a trench 420 yards long, 5 wide, and 3 deep ?

The work done in the first instance $= 210 \times 3 \times 2$.

„ second „ $= 420 \times 5 \times 3$.

The hours the 252 men work $= 55$ hours.

And we shall find the hours the 22 men are employed, and afterwards deduce the days by dividing by 9.

$$\left.\begin{array}{ccc} 22 & : & 252 \\ 210 \times 3 \times 2 & : & 420 \times 5 \times 3 \end{array}\right:: 55.$$

$$\therefore \frac{252 \times 420 \times 5 \times 3 \times 55}{22 \times 210 \times 3 \times 2} = 126 \times 5 \times 5 \text{ hours.}$$

$$\therefore \text{days} = \frac{126 \times 5 \times 5}{9} = 14 \times 25 = 350 \text{ days.}$$

EXAMPLES.

(1.) If the wages of 20 labourers for 6 months amount

to 120*l.*, what will the wages of 23 labourers for 7 months?

(2.) If 8 men can build 32 yards of wall in 3 days, how many men can in 4 days build 96 yards?

(3.) If the carriage of 4 cwt. 3 qrs. 130 miles be 2*l.* 11*s.* $5\frac{1}{2}$*d.*, what will be the carriage of 17 cwt. 2 qrs. 95 miles?

(4.) A family of 11 persons spend 352*l.* 16*s.* in 7 months; how much would 24 persons, living at the same rate, expend in 9 months?

(5.) If 1,000 soldiers consume 650 quarters of wheat in 245 days, how many quarters will 1,275 men eat in 365 days?

(6.) If a barrel of beer be sufficient for a family of 8 persons 20 days, how many barrels will a family of 12 persons require in the course of a year?

(7.) If 18 horses will consume 9 bushels of oats in 5 days, how many horses would consume 36 bushels in 12 days?

(8.) If 28 men can mow 150 acres of grass in 14 days, how many acres will 42 men mow in 18 days.

(9.) If 450*l.* gain 9*l.* 10*s.* in 186 days, what will be the gain of 750*l.* in a year.

(10.) If 1,000*l.* gain 35*l.* in 10 months, what sum of money will gain 36*l.* in 11 months.

(11.) If a man can walk 280 miles in 14 days, walking 8 hours each day, what distance can a person walk in 15 days, walking 9 hours a-day?

(12.) If 56 men can dig a trench 100 feet long, 4 wide, and 3 deep, in 12 days, working 10 hours each day, how many men can dig a trench 4 times greater in every respect, working 32 days, 8 hours each day.

(13.) What will be the wages of 24 men, working 8 hours a-day for 216 days, when the wages of 9 men working 10 hours a-day for 12 days amounted to 8*l.* 2*s.*

(14.) A garrison of 936 men consumes 351 quarters of wheat in 7 months; how many men will consume 1,470 quarters in 5 months?

(15.) If 24 men can build a house in 35 days, in how many days can 21 men build 3 such houses?

(16.) If 27 men earn 378*l.* in 20 weeks, in what time will 29 men earn 872*l.* 18*s.*?

(17.) If 4 oxen can be kept on 5 acres for 6 months, how many sheep can be kept on 28 acres for 4 months, if 5 sheep eat as much as 1 ox?

(18.) If a loaf of bread, weighing 4 lb. 4 oz. cost $7\frac{1}{2}d.$ when wheat is 46*s.* per quarter, what ought to be the weight of the 6*d.* loaf when wheat is 50*s.* a quarter?

(19.) If the expense of 363 paupers for 7 weeks be 423*l.* 10*s.*, what will be the cost of 456 paupers for 9 months?

(20.) If a family of 9 persons can purchase and consume annually 8 sacks of flour, (each sack weighing 280lb.,) when flour is 36*s.* a sack, and their wages 30*s.* a-week, how much flour can the same family equally afford to purchase, when flour is 40*s.* a sack, and their wages 28*s.* a-week?

(21.) If an elephant consume 100 lb. of hay in a day, how much would one which is one fourth bigger consume?

(22.) If 28 men, 14 women, and 21 boys, cut, and carry a field of wheat of 108 acres in 7 days, how many additional men will be necessary to cut and carry 120 acres in 6 days; the work done by each man, woman, and boy being proportional to 3, 2, 1?

ANSWERS

TO THE

QUESTIONS ON ARITHMETIC.

ADDITION.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
1. . . .	338	14. . . .	34195	27. . . .	477845
2. . . .	417	15. . . .	46408	28. . . .	363828
3. . . .	389	16. . . .	41944	29. . . .	288771
4. . . .	365	17. . . .	35563	30. . . .	295345
5. . . .	371	18. . . .	29600	31. . . .	405608
6. . . .	357	19. . . .	51479	32. . . .	40086
7. . . .	397	20. . . .	35349	33. . . .	315272
8. . . .	3764	21. . . .	50508	34. .	27,452,262
9. . . .	2689	22. . . .	46057	35. . .	6,201,906
10. . . .	3933	23. . . .	44545	36. . . .	34,857
11. . . .	3831	24. . . .	353178	37. . . .	8,575
12. . . .	4194	25. . . .	548078	38. . . .	164
13. . . .	5133	26. . . .	538756	39. . . .	1827

SUBTRACTION.

1. . . . 151	9. . 178668052	15. . . . 223
2. . . . 216	10. 3768432309	16. . . 9,030,232
3. . . . 118	11. 7798888880	17. . . 1,262,364
4. . . . 1888	12. . . . 771	18. . . 1,659,330
5. . . . 29878	13. 2348; 1921;	19. . . 356,808*l.*
6. . . . 349789	1491; 1004;	20. . . 360; 335
7. . . . 46612197	742; 588	21. . . 13,499,802
8. . 1934080829	14. . . . 55*l.*	22. . . . 40

MULTIPLICATION.

Ex.	Ans.	Ex.	Ans.
1....	46913578	29....	63827681472
2....	958166912	30....	43487063964
3....	147248690	31....	1188
4....	725001598	32...	3916; 15092;
5....	170369208		19008
6....	1177777449	33....	3653
7....	144958428	34....	10353938179
8....	1881597048	35....	9711141608
9....	212757788	36....	13498430243
10....	3703699344	37....	7953088488
11....	181217285	38....	30158573534
12....	44868024155	39....	12,671,485,616
13....	366904098	40....	47329472710
14....	7491894144	41....	31711200662
15....	3045006237	42....	72260019489
16....	53530358924	43....	76286260518
17....	633364064	44....	22560990496
18....	2883526104	45....	27587451575
19....	3865557376	46....	137573763540
20....	20768386536	47....	107831655984
21....	2813110515	48....	227036473784
22....	24479346168	49....	345603314905
23....	4787536260	50....	121010919488
24....	89763425000	51....	259121299826
25....	5844986774	52....	334671674800
26....	25802468351	53....	441358988106
27....	4349214372	54....	149339640384
28....	91826768940	55....	481011943841

Ex.	Ans.	Ex.	Ans.
56. . . .	620191133430	68. . . .	135959687654
57. . . .	175459528278	69. . . .	461366820057
58. . . .	107305188502	70. . . .	352424226750
59. . . .	70520538216	71. . . .	570228393600
60. . . .	55865256435	72. . . .	96803000000
61. . . .	264093908584	73. . . .	445590840000
62. . . .	601151299210	74. . .	4587495413808
63. . . .	585771302158	75. . .	1194262590444
64. . . .	128122150848	76. . .	1319139442993
65. . . .	379750873216	77. . .	5322286849459
66. . . .	559755927272	78. . . .	913578000000
67. . . .	547612083301	79. . .	3201303000000

DIVISION.

1. . . .	289174634	15. . . .	391230577
2. . . .	3993199938	16. . . .	890120308
3. . . .	274524379	17. . . .	$21429216\frac{10}{11}$
4. . . .	$873711596\frac{1}{3}$	18. . . .	291930807
5. . . .	187069406	19. . . .	$475163654\frac{1}{12}$
6. . . .	1921195589	20. . . .	367620704
7. . . .	95647133	21. . . .	$57360504\frac{10}{11}$
8. . . .	218146413	22. . . .	$36583869\frac{1}{19}$
9. . . .	$59552707\frac{5}{6}$	23. . . .	$8379503\frac{6}{23}$
10. . . .	226325522	24. . . .	$12301376\frac{12}{24}$
11. . . .	$68319379\frac{6}{7}$	25. . . .	$6486033\frac{14}{46}$
12. . . .	$816152864\frac{1}{7}$	26. . . .	$18599205\frac{17}{19}$
13. . . .	$58309034\frac{6}{8}$	27. . . .	$10150494\frac{18}{37}$
14. . . .	$1208645305\frac{3}{8}$	28. . . .	$56808943\frac{33}{65}$

Ex.	Ans.	Ex.	Ans.
29. . . .	$338517\frac{24}{71}$	45. . . .	$5701609\frac{118}{128}$
30. . . .	$11281381\frac{1}{82}$	46. . . .	$8293506\frac{633}{835}$
31. . . .	$811546\frac{929}{984}$	47. . . .	$3954310\frac{603}{2345}$
32. . . .	$56426\frac{9248}{45}$	48. . . .	$1565003\frac{3146}{5729}$
33. . . .	$28302\frac{2808}{77}$	49. . . .	$7137.65\frac{234}{7283}$
34. . . .	$42354\frac{245}{952}$	50. . . .	$655366\frac{14200}{35792}$
35. . . .	$3617047\frac{7}{96}$	51. . . .	$35138\frac{3321}{7000}$;
36. . . .	$7960706\frac{17}{87}$		$49193\frac{3321}{5000}$
37. . . .	$4972643\frac{188}{123}$	52. . . .	$177165\frac{373}{2700}$;
38. . . .	$14773800\frac{56}{234}$		$38889\frac{11173}{12300}$
39. . . .	$8433012\frac{64}{542}$	53. . . .	$1487\frac{359287}{630000}$;
40. . . .	$19449806\frac{345}{476}$		$28058\frac{1781}{4400}$
41. . . .	$4835783\frac{95}{379}$	54. . . .	18
42. . . .	$140611\frac{676}{878}$	55. . . .	$43,984\frac{112}{141}$
43. . . .	$15674964\frac{240}{449}$	56. . . .	$101\frac{2284}{48595}$
44. . . .	$16361542\frac{118}{279}$		

REDUCTION.

1. . . .	12500	10. . . .	265569
2. . . .	30000	11. . . .	1232555
3. . . .	266880	12. . . .	220752
4. . . .	49480	13. . . .	1467648
5. . . .	1310880	14. . . .	1891
6. . . .	356	15. . . .	4387
7. . . .	6149	16. . . .	27540
8. . . .	30603	17. . . .	601128
9. . . .	3696	18. . . .	2505150

Ex.	Ans.
19. . . .	6832
	s. *d.*
20. . . .	1498 8
21. . . .	13917
	£ *s.* *d.*
22. . . .	107 0 3
23. . . .	54 19 $7\frac{3}{4}$
24. . . .	101164
25. . . .	42544
26. . . .	283248
27. . . .	69
28. . . .	100408
29. . . .	383096000
30. . . .	21052
31. . . .	23
32. . . .	728
33. . . .	86400; 604800; 31536000
34. . . .	267140
35. . . .	2138494
36. . . .	62961
37. . . .	2627
38. . . .	21956
39. . . .	475
40. . . .	3347
41. . . .	3356892
42. . . .	295
43. . . .	11700
	mil. yds.
44. . . .	4366 1172

Ex.	Ans.
45. . . .	5567;
	ells Flem. qr.
	9278 1
	ells Fren. qr.
	4639 1
46. . . .	872669
47. . . .	8735
	hds. gall. qt.
48. . . .	175 4 1
49. . . .	162103
50. . . .	28840
51. . . .	156
52. . . .	56940
53. . . .	$5626\frac{11}{42}$
54. . . .	$6564\frac{22}{36}$
55.	1082; 1262; 154; 179
56. . . .	633600
57. . . .	44000000
58. . . .	88
	yds. ft. in.
59. . . .	51 0 $2\frac{3172}{3616}$
	mil. yds. ft. in.
60. . . .	1 1333 2 3
	min. sec.
61. . . .	8 20
	yrs. dys. hrs.
62. . . .	8 13 8
	wks. dys. hrs.
63. . . .	2 6 20
	com. yrs. dys.
64. . . .	219 65

COMPOUND ADDITION.

Ex.	Ans.		
	£	s.	d.
1. . . .	140	13	11
2. . . .	144	12	$1\frac{1}{3}$
3. . . .	294	11	$4\frac{1}{4}$
4. . . .	342	15	$9\frac{1}{2}$
5. . . .	746	0	$11\frac{1}{4}$
6. . . .	1032	15	$10\frac{3}{4}$
7. . . .	904	11	6
8. . . .	250	17	2
9. . . .	381	0	5
10. . . .	3545	19	$0\frac{1}{2}$
11. . . .	2831	11	$2\frac{1}{4}$
12. . . .	2074	15	7

	tons.	cwt.	qr.	lbs.
13. . . .	5	18	0	19

	cwt.	qr.	lbs.	oz.	dr.
14. . . .	1	1	5	4	5

	tons.	cwt	qrs.
15. . . .	3332	7	2

	lb.	oz.	dwt.	gr.
16. . . .	73	5	8	5

	tons.	cwt.	qr.	lbs.
17. . . .	7	17	0	15

	ac.	ro.	po.
18. . . .	295	0	36

Ex.	Ans.		
	mil.	fur.	yds.
19. . . .	221	2	135

	fur.	yds.	ft.	in.
20. . . .	3	24	1	2

	wk.	dys.	hrs.	min.	sec.
21. . . .	1	6	21	38	59

	lbs.	oz.	dr.	sc.
22. . . .	125	3	1	0

	lbs.	oz.	dr.	sc.	gr.
23. . . .	6	10	5	1	2

	E. ells.	qr.	nails.
24. . . .	291	0	3

	yds.	qrs.	nails.
25. . . .	395	3	3

	qrs.	bsh.	pks.
26. . . .	326	0	3

	wks.	dys.	hrs.	min.
27. . . .	266	3	17	4

	qrs.	bsh.	pk.	gall.	qts.
28. . . .	5	2	0	1	2

	sol. yds.	ft.	in.
29. . . .	341	6	1568

	leag.	mil.	fur.
30. . . .	365	1	2

COMPOUND SUBTRACTION.

Ex.	Ans.		
	£	s.	d.
1....	7	3	5
2....	9	2	8
3....	35	17	11
4....	5	1	1¼
5....	16	17	9¾
6....	11	17	6½
7....	249	7	5¼
8....	76	12	7¾
9....	264	15	6¾
	lbs.	oz.	dwt. gr.
10....	7	9	19 7
	cwt.	qrs.	lbs.
11....	8	2	7
	lbs.	oz.	dr.
12....	14	14	14

Ex.	Ans.			
	oz.	dr.	sc.	gr.
13....	7	6	2	5
	lbs.	oz.	dr.	sc.
14....	9	10	4	2
	mil.	fur.	po.	
15....	87	5	24	
	yds.	qrs.	nails.	
16....	185	2	3	
	ac.	ro.	po.	
17....	191	3	35	
	qrs.	bush.	pks.	
18....	1733	3	3	
	dys.	hrs.	min.	
19....	99	1	42	
	hrs.	min.	sec.	
20....	197	50	41	
	sol. yds.	ft.	in.	
21....	885	13	1629	

COMPOUND MULTIPICATION.

					£	s.	d.
1....	94	7	7	7....	1885	0	2
2....	70	19	6	8....	4258	1	0
3....	723	5	5	9....	315	19	9½
4....	60	18	1¾	10....	952	15	10¾
5....	495	7	1½	11....	887	5	9
6....	293	3	11⅓	12....	83	1	4

Ex.	Ans. £	s.	d.
13. . . .	722	17	4
14. . . .	3976	9	$7\frac{1}{2}$
15. . . .	1878	2	3
16. . . .	4024	16	0
17. . . .	955	7	11
18. . . .	10193	15	$10\frac{1}{2}$
19. . . .	8713	15	0
20. . . .	3252	5	9
21. . . .	1895	17	0
22. . . .	283	5	$3\frac{1}{4}$
23. . . .	77	5	$9\frac{1}{2}$
24. . . .	1890	4	2
25. . . .	258	5	$4\frac{1}{2}$
26. . . .	82	15	$10\frac{1}{2}$
27. . . .	86	10	$7\frac{1}{2}$
28. . . .	47556	1	0
29. . . .	510	8	3
30. . . .	2221	3	$3\frac{3}{4}$
31. . . .	303	6	$3\frac{1}{4}$
32. . . .	19605	17	$11\frac{1}{4}$
33. . . .	10055	1	$10\frac{1}{2}$
34. . . .	3648	11	$10\frac{1}{2}$
35. . . .	11379	1	$7\frac{1}{4}$
36. . . .	16942	14	$11\frac{3}{4}$
37. . . .	2937	7	9
38. . . .	37403	15	$7\frac{1}{2}$
39. . . .	37296	14	$0\frac{3}{4}$

Ex.	Ans.			
	tons.	cwt.	qr.	lb.
40. . . .	15	1	6	
	2	13	2	7
	3	8	3	13
	4	11	3	8
	mil.	fur.	yds.	
41. . . .	12	7	41	
	21	3	215	
	34	3	36	
	47	2	77	
	tons.	cwt.	qrs.	lb.
42. . . .	72	2	3	13
43. . . .	574	1	2	14
	lb.	oz.	dwt.	gr.
44. . . .	103	1	19	9
	lb.	oz.	dr.	sc.	gr.
45. . . .	1	5	1	1	9
	sc.	ro.	po.	
46. . . .	315	2	37	
	yds.	qr.	nails.	
47. . . .	9059	1	2	
	qrs.	bsh.	pks.	
48. . . .	258268	2	1	
	wks.	dys.	hrs.	min.	sec.
49. . . .	5	4	22	16	24
	yrs.	wks.	dys.	hrs.	min.
50. . . .	12	28	4	12	39

COMPOUND DIVISION.

Ex.	Ans. £	s.	d.
1....	108	16	$4\frac{1}{4}$
2....	122	12	$7\frac{1}{2}$
3....	12	5	$5\frac{1}{2}$
4....	58	9	$9\frac{3}{4}$
5....	62	19	8
6....	9	16	$11\frac{3}{4}$
7....	23	18	$10\frac{1}{4}$
8....	26	6	7
9....	27	3	$3\frac{1}{2}$
10....	0	18	$0\frac{3}{4}$
11....	47	7	7
12....	94	15	$2\frac{3}{4}$
13....	53	16	$11\frac{1}{2}$
14....	24	2	$10\frac{1}{2}$
15....	36	16	$4\frac{1}{2}$
16....	75	9	$7\frac{1}{4}$
17....	123	0	$11\frac{1}{2}$
18....	61	10	$5\frac{3}{4}$
19....	13	13	9
20....	76	4	$8\frac{1}{4}$
21....	27	3	$3\frac{1}{2}$
22....	4	9	$8\frac{3}{4}$
23....	7	3	$10\frac{1}{4}$
24....	15	0	$9\frac{1}{4}$
25....	35	0	$7\frac{1}{4}$
26....	9	8	$10\frac{3}{4}$
27....	1	0	$9\frac{1}{2}$
28....	1	15	$5\frac{1}{4}$

Ex.	Ans. £	s.	d.
29....	2	9	8
30....	50	6	$4\frac{1}{4}$
31....	97	0	$4\frac{1}{2}$
32....	2	3	4
33....	13	8	5
34....	7	9	$3\frac{1}{2}$
35....	4	0	$3\frac{1}{2}$
36....	9	15	$4\frac{1}{4}$
37....	9	5	$6\frac{1}{4}$
38....	1	7	$7\frac{1}{4}$
39....	7	0	$0\frac{3}{4}$
40....	0	3	$5\frac{3}{4}$
41....	0	11	$5\frac{3}{4}$
42....	4	2	3
43....	8	3	$3\frac{1}{4}$
44....	7	13	$7\frac{1}{2}$
45....	3	1	$10\frac{1}{8}$
46....	4	16	$9\frac{1}{2}$
47....	6	1	$2\frac{3}{4}$
48....	1	5	0

	cwt.	lb.	oz.
49....	7	13	6

	mil.	yds.	ft.	in.
50....	64	490	0	9

	ton.	cwt.	qrs.	lb.
51....	1	17	3	18

	ac.	ro.	po.	yds.	ft.	in.
52...	1	0	1	18	8	$118\frac{30}{31}$

Ex	Ans.				
		yds.	qrs.	nails.	
53. . . .		12	0	$2\frac{2}{4}\frac{1}{7}$	
	mon.	wks.	dys.	hrs.	min.
54. . . .	2	2	0	14	42
55. . . .			28		
56. . . .			42		
57. . . .			35		
58. . . .			117		
59. . . .			29		
60. . . .			23		

Ex.	Ans.			
61. . . .		31		
62. . . .		63		
	£	s.	d.	
63. . . .	1	16	8	
64. . . .		47		
	cwt.	qrs.	lb.	
65. . . .	23	2	27	
	£	s.	d.	f
66. . . .	2	3	1	$+\frac{3}{3}\frac{0}{5}$
67. . . .	0	5	$3\frac{3}{4}$	
68. . . .		83		

PRACTICE.

	£	s.	d.
1. . . .	143	6	0
2. . . .	68	7	6
3. . . .	144	1	0
4. . . .	57	16	0
5. . . .	90	13	3
6. . . .	98	4	0
7. . . .	77	2	2
8. . . .	54	19	6
9. . . .	37	1	9
10. . . .	51	9	$10\frac{1}{2}$
11. . . .	198	13	7
12. . . .	56	11	7
13. . . .	11	0	$4\frac{1}{2}$
14. . . .	6	0	$5\frac{3}{4}$

	£	s.	d.
15. . . .	26	14	9
16. . . .	93	4	6
17. . . .	24	11	0
18. . . .	71	3	$5\frac{1}{4}$
19. . . .	24	8	$0\frac{1}{4}$
20. . . .	53	16	$6\frac{1}{3}$
21. . . .	20	1	$5\frac{3}{4}$
22. . . .	80	1	$9\frac{3}{4}$
23. . . .	49	13	4
24. . . .	67	10	$8\frac{1}{2}$
25. . . .	18	8	$0\frac{3}{4}$
26. . . .	46	1	1
27. . . .	74	8	$5\frac{1}{4}$
28. . . .	128	10	$10\frac{3}{4}$

Ex.	Ans. £	s.	d.
29. . . .	52	8	$1\frac{1}{2}$
30. . . .	78	6	$3\frac{3}{4}$
31. . . .	121	10	10
32. . . .	177	19	2
33. . . .	56	5	$8\frac{1}{4}$
34. . . .	172	15	$4\frac{1}{2}$
35. . . .	223	12	$6\frac{1}{2}$
36. . . .	116	0	$3\frac{3}{4}$
37. . . .	156	8	$1\frac{1}{2}$
38. . . .	221	16	$5\frac{1}{4}$
39. . . .	103	4	5.
40. . . .	227	0	1.
41. . . .	248	8	8
42. . . .	145	15	0
43. . . .	243	5	$8\frac{1}{2}$
44. . . .	199	19	4
45. . . .	160	9	$11\frac{1}{4}$
46. . . .	43	7	0
47. . . .	251	1	$0\frac{1}{2}$
48. . . .	179	6	6
49. . . .	351	6	8
50. . . .	118	4	1
51. . . .	426	13	0
52. . . .	138	9	$7\frac{1}{4}$
53. . . .	259	14	$11\frac{1}{4}$
54. . . .	224	19	$0\frac{1}{2}$
55. . . .	203	1	$8\frac{1}{4}$
56. . . .	1099	2	10
57. . . .	2084	3	3
58. . . .	369	15	0

Ex.	Ans. £	s.	d.
59. . . .	271	7	$0\frac{1}{2}$
60. . . .	419	18	$3\frac{1}{4}$
61. . . .	383	11	$9\frac{1}{4}$
62. . . .	2799	13	10
63. . . .	1774	3	$5\frac{1}{4}$
64. . . .	2214	19	$7\frac{3}{4}$
65. . . .	1130	9	$0\frac{3}{4}$
66. . . .	2590	7	10
67. . . .	184	9	$0\frac{1}{4}$
68. . . .	186	1	$2\frac{3}{4}$
69. . . .	285	8	2
70. . . .	282	14	$9\frac{3}{4}$ *f.*
71. . . .	273	12	$10\frac{3}{4} + \frac{1}{2}$
72. . . .	292	12	$2\frac{3}{4}$
73. . . .	1381	10	0
74. . . .	978	0	0
75. . . .	2317	13	4
76. . . .	2660	13	4
77. . . .	4115	0	0
78. . . .	1433	15	0
79. . . .	2162	0	0
80. . . .	512	8	0
81. . . .	374	12	0
82. . . .	1539	3	4
83. . . .	1261	3	4
84. . . .	446	0	0
85. . . .	321	10	0
86. . . .	616	10	0
87. . . .	572	18	0
88. . . .	765	8	0

Ex.	Ans. £	s.	d.
89. . .	236	18	4
90. . .	298	10	0
91. . .	380	11	0
92. . .	712	19	0
93. . .	2476	4	0
94. . .	2326	10	0
95. . .	1723	1	0
96. . .	3249	13	6
97. . .	2933	4	0
98. . .	1423	12	0
99. . .	2009	14	0
100. . .	328	4	11
101. . .	2675	15	0
102. . .	1525	3	0
103. . .	745	4	0
104. . .	2937	12	0
105. . .	1104	7	0
106. . .	4267	5	0
107. . .	3464	6	0
108. . .	6928	12	0
109. . .	2272	10	0
110. . .	3408	15	0
111. . .	2585	12	0
112. . .	5172	0	0
113. . .	4378	7	0
114. . .	7899	1	0
115. . .	3296	14	0
116. . .	6504	6	0
117. . .	5486	5	0
118. . .	7921	2	0

Ex.	Ans. £	s.	d.
119. . .	149	3	6
120. . .	504	8	2
121. . .	106	15	0
122. . .	428	17	4
123. . .	364	8	9
124. . .	113	8	9
125. . .	313	6	0
126. . .	415	4	9
127. . .	433	8	4
128. . .	601	7	9
129. . .	324	7	4
130. . .	392	3	10
131. . .	1229	19	9
132. . .	1512	7	9
133. . .	692	10	8
134. . .	1615	4	2
135. . .	633	13	6
136. . .	1202	10	0
137. . .	272	5	9
138. . .	666	10	8
139. . .	521	3	1
140. . .	289	17	6
141. . .	1005	3	6
142. . .	454	16	6
143. . .	854	12	6
144. . .	1001	12	0
145. . .	1954	15	10
146. . .	209	18	3
147. . .	630	3	3
148. . .	1443	15	0

Ex.	Ans. £	s.	d.
149. . .	181	6	8
150. . .	1166	19	2
151. . .	1875	19	1
152. . .	1132	12	0
153. . .	2244	2	9
154. . .	995	3	8
155. . .	325	6	6
156. . .	1725	10	7
157. . .	890	3	3
158. . .	1287	4	3
159. . .	1448	8	6
160. . .	4236	12	0
161. .	12678	15	0
162. .	42306	6	8
163. . .	5080	16	8
164. .	19530	11	8
165. .	29137	10	0
166.	124817	6	8
167. . .	7266	14	$3\frac{1}{2}$
168. .	13449	1	4
169. .	16870	13	0
170. .	76736	9	2
171. .	20746	2	3
172. .	35142	0	3
173. .	32311	17	8
174. . .	5923	18	9

Ex.	Ans. £	s.	d.	f.
175. . .	7102	1	$7\frac{1}{2}$	
176. . .	7970	8	4	
177. . .	9395	18	6	
178. . .	6913	17	6	
179. .	14047	14	$11\frac{3}{4}$	
180. .	85344	3	9	
181. .	45993	6	$6\frac{3}{4}$	
182. .	12480	0	6	
183. . .	70	14	6	
184. . .	194	10	$10\frac{1}{2}$	$+\frac{6}{7}$
185. . .	143	13	$5\frac{1}{4}$	$+\frac{1}{14}$
186. . .	87	14	$9\frac{1}{2}$	$+\frac{1}{4}$
187. . .	75	18	$0\frac{3}{4}$	$+\frac{3}{10}$
188. . .	1070	15	$7\frac{1}{2}$	
189. . .	3	16	$11\frac{1}{2}$	$+\frac{2}{5}$
190. . .	4	10	$8\frac{1}{2}$	
191. . .	127	5	$1\frac{1}{2}$	
192. . .	236	2	$7\frac{3}{4}$	$+\frac{1}{3}$
193. . .	66	17	$5\frac{1}{4}$	$+\frac{5}{8}$
194. . .	20	14	$9\frac{1}{4}$	
195. . .	24	19	11	$+\frac{1}{2}$
196. . .	47	0	$5\frac{1}{2}$	$+\frac{1}{2}$
197. .	13510	17	6	
198. . .	92	18	$1\frac{1}{2}$	
199. . .	272	4	$4\frac{1}{2}$	
200. . .	5331	0	$3\frac{3}{4}$	

PROPORTION.

Ex.	Ans. £	s.	d.	f.
1. . . .	10	3	6	
2. . . .	46	9	$11\frac{1}{4} + \frac{3}{13}$	
3. . . .	1292	11	$10\frac{1}{2}$	
4. . . .	9	10	0	
5. . . .	2	10	$7\frac{1}{2}$	
6. . . .	33 men.			
	£	s.	d.	f.
7. . . .	343	0	$6\frac{1}{2} + \frac{1}{5}$	
8. . . .	650	0	0	
9. . . .	6*d.*			
	£	s.	d.	
10. . . .	619	16	3	
11. . . .	202	6	3	
12. . . .	210 acres.			
	£	s.	d.	
13. . . .	10	8	4	
14. . . .	92 yards.			
	£	s.	d.	f.
15. . . .	16	10	$9\frac{1}{4} + \frac{1}{2}$	
16. . . .	400 men.			
17. . . .	£12,500			
	£	s.	d.	
18. . . .	11	16	10	
19. . . .	160 yards.			
20. . . .	3072 bricks.			
	£	s.	d.	
21. . . .	301	17	6	
22. . . .	2400	12	6	
				f.
23. . . .	9	1	$2\frac{3}{4} + \frac{13}{73}$	

Ex.	Ans. £	s.	d.	f.
24. . . .	65 yards.			
	£	s.	d.	
25. . . .	169	9	$4\frac{1}{2}$	
26. . . .	80 yards.			
27. . . .	$20\frac{2}{5}$ days.			
28. . . .	208 quarters.			
	hours.		min.	
29. . . .	10		30	
	£	s.	d.	
30. . . .	0	5	9	
31. . . .	37	2	6	
32. . . .	1795	0	0	
33. . . .	0	16	3	
34. . . .	130 quarters.			
	yds.	qrs.	na.	
35. . . .	25	2	$1\frac{1}{13}$	
	£	s.	d.	
36. . . .	6	17	6	
37. . . .	8 casks.			
	£	s.	d.	f.
38. . . .	8	12	$1\frac{1}{4} + \frac{7}{33}$	
	qrs.	bush.	pks.	
39. . . .	370	2	$3\frac{2}{2}\frac{3}{7}$	
	£	s.	d.	
40. . . .	33	16	8	
41. . . .	$97\frac{1}{7}$ men.			
42. . . .	49 oxen.			
	£	s.	d.	f.
43. . . .	5	12	$9\frac{3}{4} + \frac{3}{10}$	
	wks.	day.		
44. . . .	57	1		

Ex.	Ans.
	oz. dr.
45. . . .	9 $7\frac{3}{11}$
46. . . .	$204\frac{2}{3}$ days.
	hrs. min. sec.
47. . . .	1 30 $52\frac{1}{3}$
	£ s. d.
48. . . .	103 5 0
49. . . .	4085 6 9
50. . . .	50 0 0
51. . . .	425 2 6
52. . . .	156 1. $10\frac{1}{2}$
53. . . .	27 11 3
54. . . .	$11\frac{4}{7}$ men.
	mon. wk.
55. . . .	6 1
56. . . .	$8\frac{1}{4}$ days.
57. . . .	$24\frac{4}{9}$ men.
	£ s. d. f.
58. . . .	1 14 $3\frac{1}{2} + \frac{1}{3}\frac{1}{2}\frac{8}{9}$
	yds. ft.
59. . . .	833 1
	£ s. d.
60. . . .	6 1 6

Ex.	Ans.
	tons. cwt. qrs. lbs. oz.
61. . . .	61 2 2 16 4;
	£ s. d.
	568 6 3
62. . . .	367 10 0. f.
63. . . .	64 10 $5\frac{1}{2} + \frac{1}{2}$
	mil. yds.
64. . . .	15 $1389\frac{2}{5}\frac{7}{7}$;
	£ s. d.
	3 12 $0\frac{1}{2}$
65. . . .	450 ells Eng.
	£ s. d. f.
66. . . .	682 14 $8\frac{3}{4} + \frac{1}{1}\frac{1}{9}$
67. . . .	15;
	£ s. d.
	18 2 6
	francs. centimes.
68. . . .	6461 70
	yds. qrs.
69. . . .	1158 $2\frac{6}{4}$
	s. d.
70. . . .	6 9

COMPOUND PROPORTION.

1. . . .	£161
2. . . .	18 men.
	£ s. d. f.
3. . . .	6 18 $6\frac{1}{2}$
4. . . .	989 13 $5\frac{1}{4} + \frac{0}{1}\frac{}{1}$

	qrs. bush. pks.
5. . . .	1234 5 $1\frac{1}{4}\frac{8}{9}$
6. . . .	$27\frac{3}{8}$ barrels.
7. . . .	30 horses.
	ac. ro. po.
8. . . .	289 1 $5\frac{4}{7}$

Ex.	Ans. £	s.	d.	f.
9. . . .	31	1	$4\frac{5}{4} + \frac{82}{93}$	
10. . . .	935	1	$3\frac{1}{2} + \frac{26}{77}$	
	mil.		yds.	
11. . . .	337		880	
12. . . .	1680 men.			
	£	s.	d.	f.
13. . . .	311	0	$9\frac{1}{2} + \frac{2}{3}$	
14. . . .	5488 men.			
15. . . .	120 days.			

Ex.	Ans.
16. . . .	43 weeks.
17. . . .	168 sheep.
	lb. oz.
18. . . .	$3 \quad 2\frac{6}{125}$
19. . . .	£2736
	sacks. lbs.
20. . . .	$6 \quad 201\frac{3}{5}$
21. . . .	195 lbs. 5 oz.
22. . . .	$13\frac{11}{81}$ men.

R. CLAY, PRINTER, BREAD STREET HILL.

ARITHMETIC:

FOR

THE USE OF SCHOOLS.

WITH NUMEROUS EXAMPLES.

BY THE REV. T. G. HALL, M.A.

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CHAPTER VI.

VULGAR FRACTIONS.

LV. A FRACTION is any part or parts of an integer or whole.

The integer is also called an Unit.

Suppose the unit to be one foot; then, since 12 inches make one foot, an inch is the twelfth part of a foot; 5 inches, the 5-twelfths part of a foot, and so on; which are written $\frac{1}{12}$, $\frac{5}{12}$. And as different numbers expressing inches may be added together, and make a number equal to their sum; or one set of inches may be subtracted from another set, and leave a remainder, which is a number of inches; or one set may be increased by any number of equal sets; or one set may be separated into a number of equal sets; so we see that fractional units may be treated as ordinary units, and the rules of Addition, Subtraction, Multiplication, and Division, may be applied to them, in the same way as units have been treated in the previous pages.

LVI. A Vulgar or Common Fraction is expressed by two numbers, one placed above the other, and a line is drawn between them.

The upper number is called the Numerator, the lower number the Denominator.

Thus in the fraction $\frac{5}{7}$, or five-sevenths, 5 is the numerator, and 7 is the denominator; and the fraction means

that the unit has been divided into 7 parts, and 5 of those parts have been taken.

A Proper Fraction is one whose numerator is less than its denominator, as $\frac{2}{3}$, $\frac{7}{12}$.

An Improper Fraction is one whose numerator is not less than its denominator, as $\frac{5}{5}$, $\frac{8}{7}$.

A Compound Fraction is a fraction of a fraction, as $\frac{1}{2}$ of $\frac{1}{3}$; $\frac{3}{4}$ of $\frac{5}{6}$.

A Mixed Number consists of a whole number and a fraction, as $7\frac{2}{5}$, which signifies 7 integers and two fifths of an integer.

A Complex Fraction is one that has a fraction or mixed number in either or both its members, as $\frac{\frac{2}{3}}{4}$, $\frac{7\frac{1}{2}}{3\frac{1}{4}}$, $\frac{2}{5\frac{1}{2}}$.

Every whole number may be expressed as a fraction whose denominator is 1; thus 4 is $\frac{4}{1}$.

LVII. We have hitherto considered a simple fraction as indicating by its denominator the number of parts into which the unit has been divided, and by the numerator, the number of those parts that are taken. But we may also say that the numerator is the number of units taken, and the denominator is the divisor of those units; and then the fraction will express the number of times the denominator is contained in the fractional or subordinate units of the numerator.

Thus the fraction $\frac{2}{3}$, if the unit be a foot, is 8 inches; for $\frac{1}{3}$ of one foot is 4 inches, and $\frac{2}{3}$ of a foot is 8 inches; or we may say, $\frac{2}{3}$ is $\frac{1}{3}$ of 2 feet, or $\frac{1}{3}$ of 24 inches, or 8 inches, as before.

Hence a fraction may be considered as the quotient, arising from the division of the numerator by the denominator.

LVIII. To multiply a fraction by an integer, multiply the numerator by the integer, and retain the denominator. Thus $\frac{3}{14}$ multiplied by 4, is $\frac{12}{14}$; for the integer being divided in 14 equal parts, the second fraction must contain 4 times as many equal parts as the first; and as the first fraction contains 3 of those equal parts, the second fraction contains 12 of these equal parts.

LIX. To divide a fraction by an integer, multiply the denominator by the integer, and retain the numerator. Thus $\frac{3}{4}$ divided by 2 is $\frac{3}{8}$; for the unit being divided into twice as many parts in the latter case as in the former, each part of the latter is therefore half the magnitude of each part of the former, and as the number taken is the same, the latter fraction is half the former one.

If the numerator be divisible by the integer, divide the numerator by it, and retain the denominator. Thus $\frac{4}{5}$ divided by 2 is $\frac{2}{5}$, for $\frac{2}{5}$ obviously contains only half the number of fractional units that $\frac{4}{5}$ does.

LX. If the numerator and denominator of a fraction be both multiplied and divided by the same number, its value is unaltered; for if the numerator be multiplied by a number, the fraction is multiplied by that number; and if the denominator be multiplied by the same number, the fraction is divided by it; and if a quantity be multiplied and divided by the same number, its value is not altered. Thus $\frac{2}{3} = \frac{8}{12} = \frac{80}{120}$. Similarly, if the numerator and denominator be divided by the same number, its value is not altered, for $\frac{80}{120} = \frac{8}{12} = \frac{2}{3}$.

REDUCTION OF VULGAR FRACTIONS.

LXI. Reduction of fractions is the method by which they are changed from one denomination to another.

To reduce a whole number to a fraction with a given denominator:—

Rule.—Multiply the number by the given denominator, and the product will be the numerator of the fraction, the denominator being the given number.

EXAMPLES.

(1.) Reduce 5 to a fraction with denominator 6.

This is $\frac{5 \times 6}{6} = \frac{30}{6}$; for 5 may be considered as a fraction $= \frac{5}{1}$, and if its numerator and denominator be both multiplied by 6, its value remains unaltered.

(2.) Reduce 2, 5, 8, to fractions, with denominator 7.

(3.) Reduce 15 and 81 to fractions, with denominator 11.

(4.) Reduce 18, 32, 41, 112, to fractions, with denominator 21.

LXII. To reduce a mixed number to an improper fraction.

Rule.—Multiply the integral part by the denominator of the fractional part, add the product to the numerator of the fraction, for a new numerator, and retain the denominator.

EXAMPLES.

(5.) Reduce $3\frac{2}{5}$ to an improper fraction.

By the Rule, $3\frac{2}{5} = \frac{15+2}{5} = \frac{17}{5}$; for $3 = \frac{15}{5}$, and therefore $3\frac{2}{5} = \frac{15}{5} + \frac{2}{5} = \frac{17}{5}$.

(6.) Reduce $12\frac{3}{7}$ to an improper fraction.

$$12\frac{3}{7} = \frac{84+3}{7} = \frac{87}{7}.$$

(7.) Reduce $5\frac{5}{6}$, $14\frac{3}{9}$, $21\frac{11}{12}$, to improper fractions.

(8.) Reduce $37\frac{2}{3}$, $17\frac{7}{9}$, $29\frac{4}{7}$, to improper fractions.

(9.) Reduce $10\frac{211}{7}$, $112\frac{7}{16}$, to improper fractions.

(10.) Reduce $12\frac{5}{12}$, $314\frac{5}{16}$, to improper fractions.

(11.) Reduce $360\frac{10}{17}$, $4\frac{1682}{1974}$, to improper fractions.

(12.) Reduce $387\frac{28}{111}$, $969\frac{101}{234}$, to improper fractions.

LXIII. To reduce an improper fraction to a whole or mixed number:—

Divide the numerator by the denominator, the quotient is the integral part, and make the remainder, if any, the numerator of the fractional part, and the divisor its denominator.

EXAMPLES.

(13.) $\frac{48}{8} = 6$. (14.) $\frac{47}{8} = 5\frac{7}{8}$.

(15.) Reduce $\frac{385}{25}$ and $\frac{230}{17}$ to mixed numbers.

(16.) Reduce $\frac{3827}{21}$ and $\frac{1267}{22}$ to mixed numbers.

(17.) Reduce $\frac{5854}{59}$ and $\frac{2217}{234}$ to mixed numbers.

(18.) Reduce $\frac{10373}{29}$ and $\frac{4076391}{2019}$ to mixed numbers.

LXIV. To reduce a compound fraction to an equivalent simple fraction.

Rule.—If any of the proposed quantities be integral or mixed numbers, reduce them to improper fractions.

Then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

If the same factor or factors occur both in the numerator and in the denominator, cancel before multiplying.

EXAMPLES.

(19.) Reduce $\frac{2}{3}$ of $\frac{4}{5}$ to a simple fraction.

By the Rule, $\frac{2}{3}$ of $\frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$.

For $\frac{4}{5} = \frac{12}{15}$, and $\frac{1}{3}$ of $\frac{4}{5}$ will $= \frac{1}{3}$ of $\frac{12}{15} = \frac{4}{15}$;

and $\therefore$ $\frac{2}{3}$ of $\frac{4}{5}$ will be twice as great as $\frac{1}{3}$ of $\frac{4}{5}$, that is, it will be $= \frac{8}{15}$.

(20.) Reduce $\frac{3}{4}$ of $\frac{5}{9}$ of $2\frac{1}{3}$ to a simple fraction.

$$2\tfrac{1}{3} = \frac{7}{3}.$$

$$\therefore \frac{3}{4} \text{ of } \frac{5}{9} \text{ of } 2\tfrac{1}{3} = \frac{3 \times 5 \times 7}{4 \times 9 \times 3} = \frac{5 \times 7}{4 \times 9} = \frac{35}{36}.$$

(21.) Also $\frac{1}{2}$ of $\frac{1}{4}$; $\frac{2}{3}$ of $\frac{7}{8}$; $\frac{4}{7}$ of $\frac{5}{11}$ of $\frac{14}{25}$.

(22.) Also, $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{7}{12}$; $\frac{3}{10}$ of $\frac{4}{11}$ of $1\frac{5}{8}$; $2\frac{5}{9}$ of $3\frac{7}{12}$.

(23.) Also, $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{24}{9}$; $\frac{11}{8}$ of $\frac{10}{13}$ of $9\frac{3}{4}$.

THE GREATEST COMMON MEASURE.

LXV. When a number is contained an exact number of times in another number, the former is said to be a Measure of the latter. Thus 4 is said to be a measure of 36, since it is contained 9 times in 36.

When a number will exactly divide each of two other numbers, the first is called a Common Measure of the other two.

Thus 4 is a common measure of 12 and 16; and it is called the Greatest Common Measure, when it is the greatest divisor of both numbers: thus, 8 is the greatest common measure of 24 and 32; for it is contained in 24 3 times; and in 32 4 times; and 3 and 4 have no common divisor.

Thus also, 4 is the greatest common measure of 12 and 16.

LXVI. To find the greatest common measure of two numbers.

Rule.—Divide the greater number by the less, and the preceding divisor by the remainder, and so on, continually, till nothing be left; the last divisor is the greatest common measure.

The proof of this Rule is given in Algebra.

EXAMPLES.

(24.) Find the greatest common measure of 252 and 364.

$$
\begin{array}{r|l}
252 & 364 \\
 & 252 \\
\hline
112 & 252 \\
 & 224 \\
\hline
28 & 112 \\
 & 112 \\
\hline
\end{array}
\qquad
\begin{array}{l}
(1 \\
(2 \\
(4
\end{array}
$$

∴ 28 is the greatest common measure.

Find the greatest common measures of—

(25.) 48 and 272; 192 and 576.

(26.) 825 and 960; 1344 and 1536.

(27.) 5184 and 6012; 1408 and 1664.

(28.) 7631 and 26415; 42237 and 75582.
(29.) 6441 and 10283; 3761034 and 1081.
(30.) 2125 and 11390; 315911 and 350387.

LXVII. To find the greatest common measure of three numbers.

Rule.—To find the greatest common measure of three numbers: first find the G. C. M. of any two of the numbers, and then find the greatest common measure of that G. C. M. and of the third number; it will be the G. C. M. of the three numbers.

Ex. (31.) Find the G. C. M. of 18, 24 and 32.

Then 6 is the G. C. M. of 18 and 24.

And 2 is the G. C. M. of 6 and 32.

$\therefore$ 2 is the G. C. M. of 18, 24 and 32.

And $18 = 2 \times 9$; $24 = 2 \times 12$; $32 = 2 \times 16$: also 9, 12 and 16 have no common divisor.

In a similar manner may the G. C. M. of 4 numbers be found.

LXVIII. Find the G. C. M. of the numerators and denominators of the fractions—

(EXAMPLES.)

(32.) $\frac{48}{272}; \frac{192}{576}; \frac{825}{960}; \frac{1344}{1536}$.

(33.) $\frac{5184}{6012}; \frac{1664}{1408}; \frac{26415}{7631}; \frac{42237}{75582}$.

(34.) $\frac{6441}{10283}; \frac{3761034}{1081}; \frac{2125}{11390}; \frac{315911}{350387}$.

(35.) $\frac{1020}{1596}; \frac{483}{675}; \frac{7631}{26415}; \frac{8398}{29393}$.

LXIX. The least common multiple of two or more numbers.

A common multiple of two or more numbers is the number which is divisible by both the numbers.

Thus, 36 is a common multiple of 9 and 4: since it is divisible both by 9 and 4.

The least common multiple of two or more numbers is the least number which is divisible by each of the numbers.

Thus the least common multiple of 2, 3, 8, and 12, is 24; that being the least number which is divisible by each of the numbers 2, 3, 8, and 12.

LXX. To find the least common multiple of two or more numbers.

Rule.—Write the given numbers in a line, first omitting those which are factors of any of the others.

Divide as many as possible by a number which measures more of the numbers than any other number will; write the quotients and undivided numbers in a new line; and continue the process, until no two quotients or undivided numbers have a common divisor.

Then the continued product of the divisors, and of the numbers in the last line, will be the least common multiple required.

EXAMPLES.

(36.) Find the least common multiple, or L. C. M. of 5, 18, 9, 2, 16: omit 9 and 2, which are contained in 18.

$$\begin{array}{r} 2\,)\,5,\quad 18,\quad 16 \\ \hline 5,\quad 9,\quad 8 \end{array};$$

$\therefore\ 2 \times 5 \times 9 \times 8 = 10 \times 72 = 720$, is the L. C. M.

(37.) Find the L. C. M. of 8, 9, 6, 12, and 3.

(38.) Find the L. C. M. of 4, 18, 12, and 8.

(39.) Find the L. C. M. of 36, 42, 100, and 450.

(40.) Find the L. C. M. of 36, 63, 84, and 28.

(41.) Find the L. C. M. of 24, 32, 57, and 76.

(42.) Find the L. C. M. of 2, 3, 4, 5, 6, 7, 8, 9, and 10.

LXXI. We may also use the following Rule when the least common multiple of two high numbers is required :—

Divide the product of the two numbers by their greatest common measure, the quotient is the L. C. M. : or what is the same thing, divide one of the numbers by the G. C. M. of the two, multiply the quotient by the other number, and the product is the L. C. M.

EXAMPLES.

(43.) Find the L. C. M. of 126 and 144.

$$\begin{array}{r} 126 \,)\, 144 \,(\, 1 \\ \underline{126} \\ 18 \,)\, 126 \,(\, 7 \\ \underline{126} \end{array}$$

$$\frac{126 \times 144}{18} = 7 \times 144 = 1008.$$

(44.) Find the L. C. M. of 5187 and 5850.

(45.) Find the L. C. M. of 6281 and 326041.

(46.) Find the L. C. M. of 14186 and 13667.

LXXII. A fraction may be reduced to lower terms, when the numerator and denominator are both divisible by the same number.

Thus $\frac{54}{72} = \frac{27}{36}$; and the second fraction is in lower terms than the first, and is derived from it by dividing both numerator and denominator by 2.

LXXIII. A fraction is reduced to its lowest terms, by dividing both the numerator and the denominator by their greatest common measure.

Ex. (47.) Reduce $\frac{1702}{1886}$ to its lowest term.

1.—To find the greatest common measure:—

$$\begin{array}{r|l} 1702 & 1886 \ (\ 1 \\ & 1702 \\ \hline 184 & 1702 \ (\ 9 \\ & 1656 \\ \hline 46 & 184 \ (\ 4 \\ & 184 \\ \hline \end{array}$$

$\therefore$ 46 is the G. C. M.

$$\begin{array}{r|l} 46 & 1702 \ (\ 37 \\ & 138 \\ \hline & 322 \\ & 322 \\ \hline \end{array} \qquad \begin{array}{r|l} 46 & 1886 \ (\ 41 \\ & 184 \\ \hline & 46 \\ & 46 \\ \hline \end{array}$$

$$\therefore \frac{1702}{1886} = \frac{37 \times 46}{41 \times 46} = \frac{37}{41}.$$

LXXIV. Reduce to their lowest terms the fractions—

(EXAMPLES.)

(48.) $\frac{27}{36}, \frac{21}{24}, \frac{18}{42}, \frac{45}{54}$.

(49.) $\frac{56}{63}, \frac{40}{48}, \frac{45}{70}, \frac{36}{39}$.

(50.) $\frac{126}{144}, \frac{168}{312}, \frac{225}{675}, \frac{462}{315}$.

(51.) $\frac{1848}{27720}, \frac{1188}{8316}, \frac{1029}{2250}, \frac{441}{1323}$.

(52.) $\frac{1856}{4466}, \frac{14186}{13667}, \frac{49561}{2442641}$.

(53.) $\frac{95469}{359784}, \frac{93208}{13786}, \frac{11050}{35581}$.

(54.) $\frac{8888}{403596}, \frac{109375}{10000000}$.

(55.) $\frac{7631}{26415}, \frac{42237}{75582}$.

(56.) $\frac{100005}{814175}, \frac{26664}{1210788}$.

LXXV. To reduce fractions with different denominators into equivalent fractions with the same denominator:—

First find the least common multiple of all the denominators for the common denominator; divide it by each of the original denominators, and multiply each numerator by the respective quotients for new numerators of the fractions required.

Ex. (57.) Reduce $\frac{7}{12}$ and $\frac{5}{18}$ to fractions, having the same denominator.

Here 36 is the least common multiple of 12 and 18.

$$12\overline{)36}(3 \quad 36 \quad 18\overline{)36}(2 \quad 36$$

$$\therefore \frac{7}{12} = \frac{7}{12} \times \frac{3}{3} = \frac{21}{36};$$

$$\frac{5}{18} = \frac{5}{18} \times \frac{2}{2} = \frac{10}{36}.$$

Any number which is a multiple of 18 and 12 would have reduced these fractions to equivalent fractions with the same denominator; but the above method reduces them to fractions in their lowest terms, with the same denominator.

Thus, since 72 is a multiple of 12 and 18,

$$\frac{7}{12} = \frac{42}{72}; \text{ and } \frac{5}{18} = \frac{20}{72}.$$

LXXVI. When the denominators of the fractions have no common measure, their least common multiple is their

product, and we may use the following Rule. Multiply each numerator by all the denominators, except its own, for a new numerator, and all the denominators together, for a common denominator.

Ex. (58.) Reduce $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}$, to fractions having the same denominator.

$$\frac{1}{2} = \frac{1}{2} \times \frac{3 \times 5 \times 7}{3 \times 5 \times 7} = \frac{105}{210}.$$

$$\frac{2}{3} = \frac{2}{3} \times \frac{2 \times 5 \times 7}{2 \times 5 \times 7} = \frac{140}{210}.$$

$$\frac{3}{5} = \frac{3}{5} \times \frac{2 \times 3 \times 7}{2 \times 3 \times 7} = \frac{126}{210}.$$

$$\frac{4}{7} = \frac{4}{7} \times \frac{2 \times 3 \times 5}{2 \times 3 \times 5} = \frac{120}{210}.$$

The above method shows the principle of the Rule, that the fractions are of the same value, though changed in form; in practice we frequently work thus,

$$\left.\begin{array}{l} 1 \times 3 \times 5 \times 7 = 105 \\ 2 \times 2 \times 5 \times 7 = 140 \\ 3 \times 2 \times 3 \times 7 = 126 \\ 4 \times 2 \times 3 \times 5 = 120 \end{array}\right\} \text{new numerators;}$$

$$2 \times 3 \times 5 \times 7 = 210 \quad \text{new denominator;}$$

and $\frac{105}{210}, \frac{140}{210}, \frac{126}{210}, \frac{120}{210}$, are the required fractions.

LXXVII. Reduce the following fractions to others with a common denominator:—

(EXAMPLES.)

(59.) $\frac{5}{6}$ and $\frac{7}{12}$. (60.) $\frac{2}{3}$ and $\frac{3}{4}$.

(61.) $\frac{4}{9}$ and $\frac{5}{27}$. (62.) $\frac{2}{5}$ and $\frac{5}{8}$.

(63.) $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$ and $\frac{1}{7}$. (64.) $\frac{1}{4}, \frac{3}{8}, \frac{4}{12}$ and $\frac{5}{16}$.

(65.) $\frac{7}{19}, \frac{5}{57}$ and $\frac{11}{24}$. (66.) $\frac{5}{69}, \frac{9}{92}$ and $\frac{11}{161}$.

(67.) $\frac{2}{3}, \frac{3}{5}, \frac{5}{7}$ and $\frac{6}{25}$. (68.) $\frac{5}{12}, \frac{3}{16}, \frac{1}{21}$ and $\frac{7}{60}$.

(69.) $\frac{5}{7}, \frac{8}{9}, \frac{11}{12}, \frac{13}{15}$ and $\frac{17}{18}$.

(70.) $\frac{20}{19}, \frac{21}{20}, \frac{7}{8}$ and $\frac{1}{95}$.

LXXVIII. By reducing two fractions to others having the same denominator, their values may be compared.

Thus $\frac{2}{3}$ and $\frac{3}{5}$, when reduced to the same denominator, become $\frac{10}{15}$ and $\frac{9}{15}$; and the fractions have the same relative value as 10 and 9 have.

LXXIX. To reduce a fraction of one denomination to a fraction of another denomination, which shall have the same value:—

First find what fraction the unit of the first denomination is of the unit of the second denomination, and multiply this fraction by the given fraction: the product reduced to its lowest terms is the fraction required.

EXAMPLES.

(71.) What fraction of 1*l.* is $\frac{3}{5}$ of 1*s.*?

Since $20s. = 1l.$ $\therefore$ $1s. = \frac{1}{20}$ of $1l.$

$\therefore$ $\frac{3}{5}$ of $1s. = \frac{3}{5} \times \frac{1}{20} = \frac{3}{100}$ of $1l.$

(72.) What fraction of 1*d.* is $\frac{4}{7}$ of 1*l.*?

$1l. = 240d.$ $\therefore$ $1l. = \frac{240}{1}$ of $1d.$

$\therefore$ $\frac{4}{7}$ of $1l. = \frac{4}{7} \times \frac{240}{1} = \frac{960}{7}$.

(73.) What fraction of 5 lb. is $\frac{3}{11}$ of 1 cwt.?

1 cwt. = 112 lb. = $\frac{112}{1}$ of 1 lb. = $\frac{112}{5}$ of 5 lb.

$\therefore$ $\frac{3}{11}$ of 1 cwt. = $\frac{3}{11} \times \frac{112}{5} = \frac{336}{55}$ of 5 lb.

(74.) What fraction of a mile is $\frac{48}{49}$ of a yard?

1 mile = 1760 yards $\therefore$ 1 yard = $\frac{1}{1760}$ of 1 mile.

$\therefore$ $\frac{48}{49}$ of a yard = $\frac{48}{49} \times \frac{1}{1760} = \frac{16 \times 3}{49 \times 16 \times 110} = \frac{3}{5390}$.

(75.) Reduce $\frac{5}{6}$ of $1d.$ to the fraction of $1l.$

(76.) Reduce $\frac{4}{7}$ of 1*s.* to the fraction of a guinea.

(77.) Reduce $\frac{3}{8}$ of 8*s.* 4*d.* to the fraction of $1l.$

(78.) Reduce $\frac{5}{12}$ of 17*s.* $2\frac{1}{4}d.$ to the fraction of $1l.$

(79.) What fraction is $\frac{2}{3}$ of 5*s.* of 4*s.* 6*d.*; and $\frac{2}{7}$ of 10*s.* of a guinea?

(80.) What fractions are $\frac{3}{7}$ of 3*s.* $2\frac{2}{11}d.$, and $\frac{2}{3}$ of $\frac{4}{5}$ of 1*s.* of $1l.$

(81.) Reduce $2\frac{1}{4}$ of 4*s.* 10*d.* to the fraction of a moidore.

(82.) Reduce $4\frac{2}{11}$ of a guinea to the fraction of a sovereign.

(83.) Reduce $\frac{7}{8}$ of half-a-crown to the fraction of 6*s.*

(84.) What fraction is $\frac{1}{4}$ of 2*s.* 3*d.* of 10*s.* 6*d.*; and $\frac{2}{5}$ of 19*s.* $11\frac{3}{4}$*d.* of 1*l.* 2*s.* 4*d.*?

(85.) Reduce $\frac{2}{5}$ of 1 lb. to the fraction of a cwt.

(86.) Reduce $\frac{2}{7}$ of a dwt. to the fraction of a lb. troy.

(87.) What fraction is $4\frac{1}{3}$ of a lb. avoirdupois of a lb. troy?

(88.) Reduce 2 furlongs, 8 poles, 18 yards to the fraction of $\frac{11}{10}$ of a mile.

(89.) Reduce 5 drs. 1 scr. to the fraction of $\frac{3}{20}$ of a lb.

(90.) What fraction of a furlong is 7 yds. $2\frac{1}{7}$ ft.?

(91.) Reduce $\frac{3}{13}$ of a month to the fraction of a day, and $\frac{5}{17}$ of a week to the fraction of a year.

(92.) A pendulum vibrates $61\frac{1}{7}$ times in a minute; what ratio does the number of its vibrations in 24 hours bear to the number of seconds in the same time?

(93.) What fraction is 19*s.* $11\frac{3}{4}$*d.* of a guinea?

(94.) The Express of the Great Western travels 53 miles in 55 minutes; what fraction is that speed of that of the earth at the equator, which travels 25,000 miles in 24 hours.

(95.) What fraction is the height of the Great Pyramid, which is 477 feet, of the highest point of the Himalaya, which is 5 miles, 2 furlongs, 116 yards, $2\frac{1}{3}$ feet high?

LXXX. To find the value of a fraction in terms of the known parts of its integer:—

Multiply the numerator by the parts of the next lower denomination of the integer, and divide the product by the

denominator, and if anything remains, proceed in the same way as before ; the quotients placed in order will be the value required.

EXAMPLES.

(96.) Find the value of $\frac{5}{7}$ of 1*l.*

$$\begin{array}{r} 5 \\ 20 \\ \hline 7\,)\,100 \\ \hline 14s.\text{—}2s. \\ 12 \\ \hline 7\,)\,24 \\ \hline 3d.\text{—}3d. \\ 4 \\ \hline 7\,)\,12 \\ \hline 1\tfrac{5}{7}f. \end{array}$$

Ans. 14*s.* 3$\frac{1}{2}$*d.* $\frac{5}{7}$*f.*

Or thus,

$$\frac{5}{7} \text{ of } 1 = \frac{5}{7} \times \frac{20}{1}^{s.} = \frac{100}{7}^{s.} = 14\ \tfrac{2}{7}^{s.};$$

$$\frac{2}{7} \text{ of } 1^{s.} = \frac{2}{7} \times \frac{12}{1}^{d.} = \frac{24}{7}^{d.} = 3\ \tfrac{3}{7}^{d.};$$

$$\frac{3}{7} \text{ of } 1^{d.} = \frac{3}{7} \times \frac{4}{1}^{f.} = \frac{12}{7}^{f.} = 1\ \tfrac{5}{7}^{f.}$$

$\therefore$ *Ans.* = 14*s.* 3$\frac{1}{2}$*d.* $\frac{5}{7}$*f.*

(97.) Find the value of $\frac{3}{14}$ of 1 cwt.

$$\frac{3}{14} \text{ of 1 cwt.} = \frac{12}{14} \text{ qrs.} = \frac{12 \times 28}{14} = 24 \text{ lb.}$$

(98.) Find the value of $\frac{4}{5}$ of a mile.

$$\frac{4}{5} \text{ of a mile} = \frac{4}{5} \times \frac{8}{1} \text{ of a furlong} = \frac{32}{5} = 6\tfrac{2}{5} \text{ furlongs;}$$

$$\frac{2}{5} \text{ of a furlong} = \frac{2}{5} \times \frac{220}{1} \text{ yards} = \frac{440}{5} = 88 \text{ yards.}$$

$\therefore$ *Ans.* = 6 furlongs 88 yards.

Find the values of—

(99.) $\frac{2}{3}$ of a crown and $\frac{5}{8}$ of a shilling.

(100.) $\frac{5}{6}$ of a moidore and $\frac{7}{20}$ of 2*l.* 10*s.*

(101.) $\frac{13}{15}$ of 1*l.* and $\frac{10}{7}$ of 7*s.* 10$\frac{1}{2}$*d.*

(102.) $\frac{2}{3}$ of $\frac{3}{4}$ of 13*s.* 4*d.* and $\frac{179}{1008}$ of a guinea.

(103.) $\frac{13}{19}$ of a moidore and $\frac{29}{80}$ of 1*l.*

(104.) $\frac{3}{5}$ of a lb. troy and $\frac{19}{28}$ of a cwt.

(105.) $\frac{3}{10}$ of a day and $\frac{49}{160}$ of a week.

(106.) $\frac{4}{9}$ of a yard and $\frac{3}{88}$ of a mile.

(107.) $\frac{11}{8}$ of a bushel and $\frac{19}{7}$ of a peck.

(108.) $\frac{5}{9}$ of an Eng. ell and $\frac{7}{18}$ of a league.

(109.) $\frac{199}{366}$ of a lb. troy and $\frac{4}{7}$ of a lb. avoirdupois.

(110.) $\frac{9}{14}$ of a cwt. and $\frac{5}{3}$ of $\frac{4}{15}$ of a ton.

(111.) $\frac{7}{36}$ of a lb. apoth. wt. and $\frac{79}{1460}$ of 365 days.

LXXXI. To reduce a given quantity to the fraction of another given quantity :—

Reduce both to the same denomination ; and take the former for the numerator, and the latter for the denominator of the required fraction.

EXAMPLES.

(112.) Reduce 8*s.* 4*d.* to the fraction of 10*s.* 6*d.*

8*s.* 4*d.* = 100 pence, 10*s.* 6*d.* = 126 pence.

$\therefore \frac{100}{126}$ is the fraction required.

Since $\frac{100}{126} = \frac{50}{63}$, the latter is the fraction in lowest terms.

What fractions are—

(113.) 5½*d.* of 2*s.* ; and 3*s.* 6*d.* of 1*l.* 7*s.* 6*d.* ?

(114.) 6*s.* 4½*d.* of 13*s.* 4*d.* ; and 8*s.* 5*d.* of a guinea ?

(115.) 2 cwt. 1 qr. 15 lb. of 5 cwt. ; and 1 qr. 13 lb. 7 oz. of a ton ?

(116.) 5 hr. 48 min. 24 sec. of a day ; and 3 days 5 hr. 25 min. of a week ?

(117.) 7 oz. 4 dwt. of a lb. troy ; and 14*s.* 8*d.* of 1*l.* 13*s.* 4*d.* ?

(118.) Reduce 2 furlongs 8 poles 16 yards to the fraction of a league.

ADDITION OF FRACTIONS.

LXXXII. *Rule.*—If the fractions have the same denominator, add the numerators together, and retain the denominator; the fraction arising will be the sum required.

But if the fractions have different denominators, reduce all the fractions to fractions with the same denominator; and then take the sum of the numerators for a new numerator, and retain the common denominator; the result will be the sum required.

(119.) Find the sum of $\frac{5}{24}$ and $\frac{11}{24}$.

$$\frac{5}{24} + \frac{11}{24} = \frac{5+11}{24} = \frac{16}{24} = \frac{2}{3}.$$

(120.) Find the sum of $\frac{2}{3}$ and $\frac{7}{8}$.

$$\frac{2}{3} = \frac{2}{3} \times \frac{8}{8} = \frac{16}{24};$$

$$\frac{7}{8} = \frac{7}{8} \times \frac{3}{3} = \frac{21}{24};$$

$$\therefore \frac{2}{3} + \frac{7}{8} = \frac{16}{24} + \frac{21}{24} = \frac{37}{24}.$$

The reason of this Rule is obvious after what has been said previously.

LXXXIII. If any of the quantities are mixed numbers, take separately the sums of the integers and fractions, and add the two results together.

Ex. (121.) Find the sum of $2\frac{2}{3}$ and $5\frac{4}{5}$.

$$\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\tfrac{7}{15};$$

$$\therefore 2\tfrac{2}{3} + 5\tfrac{4}{5} = 2 + 5 + 1 + \frac{7}{15} = 8\tfrac{7}{15}.$$

LXXXIV. Improper fractions should be reduced to mixed numbers, and complex fractions to simple ones, before the sum is taken.

Ex. (122.) Find the sum of $\frac{131}{14}$ and $\frac{2\frac{1}{5}}{6}$.

$$\frac{131}{14} = 9\tfrac{5}{14}; \quad \frac{2\frac{1}{5}}{6} = \frac{11}{30};$$

$$\frac{5}{14} + \frac{11}{30} = \frac{75}{210} + \frac{77}{210} = \frac{152}{210} = \frac{76}{105};$$

$$\therefore \quad \frac{131}{14} + \frac{2\frac{1}{5}}{6} = 9\tfrac{76}{105}.$$

LXXXV. Find the values of—

(EXAMPLES.)

(123.) $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$. (124.) $\frac{1}{3} + \frac{1}{5} + \frac{1}{7}$.

(125.) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$. (126.) $\frac{2}{21} + \frac{}{23}$.

(127.) $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{42}$.

(128.) $\frac{3}{10} + \frac{7}{40} + \frac{21}{136} + \frac{21}{190}$.

(129.) $\frac{40}{41} + \frac{41}{42} + \frac{42}{43}$.

(130.) $2\frac{3}{7} + 13\frac{7}{10} + 15\frac{1}{10}$.

(131.) $12\frac{5}{8} + 7\frac{7}{12} + 3\frac{19}{24}$.

(132.) $\frac{7}{6} + 2\frac{3}{4} + \frac{1\frac{1}{2}}{4} + \frac{2\frac{3}{4}}{3\frac{1}{4}}$.

(133.) $\frac{1}{21}$ of 1*l.* + $\frac{1}{20}$ of a guinea.

(134.) $\frac{5}{9}$ of a guinea + $\frac{3}{16}$ of 1*l.* + $\frac{5}{8}$ of a shilling.

(135.) $\frac{4}{7}$ of a cwt. + $2\frac{3}{8}$ lb. + $3\frac{1}{56}$ of an oz.

(136.) $7\frac{1}{7}$ weeks + $5\frac{2}{3}$ days + $4\frac{2}{3}$ hours.

(137.) $\frac{11}{14}$ of a guinea + $\frac{5}{12}$ of a crown + $\frac{5}{8}$ of 6*s.* 8*d.* + $\frac{3}{5}$ of 10*s.* 6*d.*

SUBTRACTION OF FRACTIONS.

LXXXVI. *Rule.*—If the fraction have the same denominator, take the difference of the numerators, and put under it the common denominator. But if the fractions have different denominators, reduce them to fractions with the same denominator; and then take the difference between the numerators, and put under it the common denominator; the result will be the difference required.

Ex. (138.) Find the difference between $\frac{8}{11}$ and $\frac{5}{11}$.

$$\frac{8}{11} - \frac{5}{11} = \frac{8-5}{11} = \frac{3}{11}.$$

(139.) Find the difference between $\frac{2}{3}$ and $\frac{5}{13}$.

$$\frac{2}{3} = \frac{2}{3} \times \frac{13}{13} = \frac{26}{39};$$

$$\frac{5}{13} = \frac{5}{13} \times \frac{3}{3} = \frac{15}{39};$$

$$\therefore \frac{2}{3} - \frac{5}{13} = \frac{26}{39} - \frac{15}{39} = \frac{11}{39}.$$

LXXXVII. EXAMPLES OF SUBTRACTION.

(140.) From $\frac{7}{8}$ take $\frac{2}{3}$; from $\frac{4}{17}$ take $\frac{7}{31}$.

(141.) From $6\frac{2}{5}$ take $3\frac{4}{7}$; from $13\frac{1}{2}$ take $7\frac{3}{4}$.

(142.) From $42\frac{3}{26}$ take $24\frac{8}{13}$; from $\frac{41}{5}$ take $\frac{39}{9}$.

(143.) From 12*l.* 8*s.* $10\frac{5}{8}$*d.* take 11*l.* 17*s.* $2\frac{3}{5}$*d.*

(144.) What is the difference between $\frac{1}{19}$ of 1*l.* and $\frac{1}{20}$ of a guinea?

(145.) Find the difference between $\frac{3}{5}$ of an oz. and $\frac{5}{8}$ of a pennyweight.

(146.) What is the difference between $\frac{3}{7}$ of half-a-guinea and $\frac{7}{8}$ of a crown?

(147.) Which is greatest, $\frac{1}{21}$ of 1*l.* or $\frac{1}{22}$ of a guinea? and find the difference.

(148.) Find the difference between $\frac{5}{12}$ of a league and $\frac{7}{10}$ of a mile.

(149.) Required the difference between $7\frac{9}{10}$ of a cwt. and 2 qrs. $17\frac{1}{4}$ lb.

MULTIPLICATION OF FRACTIONS.

LXXXVIII. *Rule.*—Multiply the numerators of the fractions for a new numerator, and the denominators for a new denominator; the result is the product required.

Ex. (150.) Multiply $\frac{2}{3}$ by $\frac{4}{7}$.

By the rule $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$.

For multiplying $\frac{2}{3}$ by $\frac{4}{7}$ is the same thing as multiplying the fraction by the integer 4, and dividing it by the integer 7; now, as we have seen, to multiply $\frac{2}{3}$ by 4, we multiply the numerator by 4; and to divide $\frac{2}{3}$ by 7, we multiply the denominator by 7; both of which operations are performed at once by the rule given above.

Observe to cancel those factors which are common both to the numerators and denominators.

Ex. (151.) Find the product of $\frac{2}{3}$, $3\frac{1}{4}$, $2\frac{1}{2}$, and $\frac{4}{3}$ of $\frac{7}{5}$.

First, $3\frac{1}{4} = \frac{13}{4}$; and $2\frac{1}{2} = \frac{5}{2}$.

Then,

$$\frac{2}{3} \times \frac{13}{4} \times \frac{5}{2} \times \frac{4}{3} \times \frac{7}{5} = \frac{13 \times 7}{3 \times 3} = \frac{91}{9} = 10\frac{1}{9}.$$

LXXXIX. EXAMPLES OF MULTIPLICATION.

Find the product of—

(152.) $\frac{4}{15}$ by $\frac{3}{16}$; and $\frac{3}{64}$ by $7\frac{1}{9}$.

(153.) $\frac{14}{15}$ by $\frac{9}{10}$; and $\frac{16}{21}$ by $\frac{35}{24}$.

(154.) $\frac{512}{1071}$ by $\frac{7}{256}$; and $\frac{1}{2}$ of $\frac{7}{3}$ by $2\frac{1}{8}$ of 3.

(155.) $\frac{3}{4}$, $\frac{4}{5}$, $2\frac{1}{8}$ and $4\frac{3}{16}$.

(156.) $\frac{9}{10}$, $1\frac{1}{2}$, $\frac{20}{27}$, $\frac{4}{5}$ and $1\frac{1}{4}$.

(157.) $\frac{8}{15}$, $\frac{7}{16}$, $\frac{5}{54}$, $\frac{11}{19}$ and $\frac{38}{22}$.

(158.) $1\frac{11}{64}$, $2\frac{2}{3}$, $\frac{8}{13}$ and $\frac{26}{25}$.

(159.) 14, $\frac{5}{6}$, $\frac{4}{5}$, 9 and $6\frac{2}{7}$.

(160.) Multiply 12*l.* 19*s.* 10*d.* by $5\frac{5}{8}$.

(161.) Multiply 216*l.* 17*s.* $8\frac{1}{2}d.$ by $23\frac{5}{12}$.

(162.) Multiply 47 cwt. 2 qrs. $8\frac{1}{5}$ lb. by $17\frac{3}{4}$.

DIVISION OF FRACTIONS.

Rule.—Invert the divisor, and proceed as in Multiplication.

Ex. (163.) Divide $\frac{2}{3}$ by $\frac{5}{7}$.

$$\therefore \text{ by Rule, } \frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15};$$

or we may reduce the fraction to a common denominator, and find the quotient of the numerator of the dividend, by that of the divisor; which will be the result required:

$$\text{thus, since } \frac{2}{3} = \frac{14}{21} \text{ and } \frac{5}{7} = \frac{15}{21}.$$

$$\therefore \frac{2}{3} \div \frac{5}{7} = \frac{14}{15}.$$

But the Rule may be thus shown to be true:

To divide $\frac{2}{3}$ by $\frac{5}{7}$ we may first divide $\frac{2}{3}$ by 5, which is $\frac{2}{15}$: but the divisor is 7 times too large, and we must therefore multiply this result by 7, to obtain the true result, which is $\therefore \frac{14}{15}$.

(164.) Divide $2\frac{3}{5}$ by $7\frac{2}{3}$.

$$2\tfrac{3}{5} \div 7\tfrac{2}{3} = \frac{13}{5} \div \frac{23}{3} = \frac{13}{5} \times \frac{3}{23} = \frac{39}{115}.$$

(165.) Divide $\frac{24}{25}$ by $\frac{7}{3}$ of $\frac{2}{5}$ or by $\frac{14}{15}$.

$$\frac{24}{52} \div \frac{14}{15} = \frac{24}{25} \times \frac{15}{14} = \frac{12}{5} \times \frac{3}{7} = \frac{36}{35}.$$

XC. EXAMPLES.

Find the quotient of—

(166.) $\frac{7}{8}$ by $3\frac{1}{2}$; and $3\frac{1}{6}$ by $9\frac{1}{2}$.

(167.) $5\frac{1}{4}$ by $\frac{8}{25}$; and $\frac{3}{100}$ by $4\frac{1}{5}$.

(168.) 100 by $7\frac{1}{7}$; and $10\frac{1}{2}$ by $\frac{3}{4}$.

(169.) $\frac{21}{25}$ by $\frac{14}{15}$; and $2\frac{1}{3\,6}$ by $4\frac{5}{14}$.

(170) Divide 512*l.* 18*s.* 6*d.* by $5\frac{3}{4}$.

(171.) Divide $\frac{5}{12}$ of a guinea by $\frac{7}{16}$ of 1*l.*

(172.) Divide 23 miles 4 furlongs 120 yards by $3\frac{1}{3}$.

(173.) Divide 18 cwt. 3 qrs. $14\frac{1}{8}$ lb. by $7\frac{1}{8}$.

(174.) Divide 27*l.* 13*s.* $4\frac{1}{4}$*d.* by 2*l.* 3*s.* $4\frac{1}{2}$*d.* $\frac{1}{8}$*f.*

(175.) Divide $\frac{7}{8}$ of 1*l.* by $\frac{9}{10}$ of a moidore.

(176.) Divide $\frac{1}{5}$ of $\frac{6}{11}$ of 2*s.* 6*d.* by $2\frac{8}{11}$.

(177.) Divide $\frac{2}{3}$ of $\frac{4}{5}$ of 10*s.* 6*d.* by $\frac{1}{4}$ of $2\frac{1}{3}$ of 5*s.*

RULE OF THREE IN FRACTIONS.

XCI. The question is stated as in the ordinary Rule of Three; all the terms are reduced to fractions, or are put under a fractional form.

Rule.—Having stated the question, invert the first term (or divisor), and multiply it by the product of the second and third terms; the total product will be the answer.

EXAMPLES.

(178.) If $\frac{5}{8}$ of a cwt. cost $4\frac{7}{9}l.$, what will $4\frac{1}{2}$ lb. cost?

$$\frac{5}{8} \text{ of cwt.} = \frac{5}{8} \times 112 = \frac{5 \times 14}{1} = \frac{70}{1}.$$

$$4\tfrac{7}{9} = \frac{43}{9}; \; 4\tfrac{1}{2} = \frac{9}{2}.$$

$$\frac{\text{lb. } 70}{1} : \frac{\text{lb. } 9}{2} :: \frac{£\ 43}{9}.$$

$$\therefore \frac{1}{70} \times \frac{9}{2} \times \frac{43}{9} = \frac{43}{140} = \frac{43}{140} \times 20 = \frac{s.\ 43}{7} = 6\tfrac{1}{7}s.$$

And $6\frac{1}{7}s. = 6s.\ 1\frac{5}{7}d. = 6s.\ 1\frac{1}{2}\ \frac{6}{7}d.$

(179.) If $\frac{3}{5}$ of a yard cost $\frac{7}{12}$ of $1l.$, what will $\frac{6}{15}$ of an English ell cost?

(180.) If $\frac{3}{16}$ of a ship cost $273l.\ 2s.\ 6d.$, what is $\frac{5}{32}$ of it worth?

(181.) If the fare to Exeter by rail be $44s.\ 6d.$, and $30s.\ 6d.$ for the first and second class passengers, how much is that per mile, the distance being $193\frac{3}{4}$ miles?

(182.) If $\frac{2}{7}$ of an estate be worth 400*l.*, what is the value of $\frac{11}{48}$ of the same?

(183.) If $26\frac{3}{4}$ yards of cloth cost 4*l.* 8*s.* $6\frac{1}{2}d.$, what is the price of $1\frac{1}{2}$ yard?

(184.) If $\frac{4}{15}$ of a guinea buy $2\frac{1}{2}$ yards of cloth, how much will $\frac{7}{21}$ of a 1*l.* buy?

(185.) If the penny loaf weigh $6\frac{7}{10}$ of an oz. when wheat is 4*s.* 6*d.* per bushel, what will be the weight when it is 7*s.* 6*d.*?

(186.) If when the days are $14\frac{5}{8}$ hours long, a man can do a certain work in $41\frac{5}{12}$ days, how long will it take him to do the same work when the days are $9\frac{3}{16}$ hours long?

(187.) A man left $\frac{3}{5}$ of his estate to his eldest son, who sold $\frac{11}{17}$ of his share for 1200*l.*, what was the value of the whole estate?

(188.) What will be the cost of 4 chests of tea, each weighing 83lb. $6\frac{2}{3}$oz., at 4*s.* $6\frac{1}{8}d.$ per lb.?

(189.) If $7\frac{1}{8}$cwt. of sugar cost $22\frac{3}{16}l.$, what will be the value of $3\frac{4}{15}$cwt.?

(190.) A silver cup weighs 1lb. $8\frac{1}{3}$oz.; what is its value when 12 spoons weighing 3lb. $0\frac{1}{12}$oz. cost 14*l.*?

CHAPTER VII.

DECIMAL FRACTIONS.

XCII. DECIMAL FRACTIONS are those which have for their denominators either ten, or products of ten by successive tens, as 100, 1,000, 10,000, &c.

Thus $\frac{3}{10}$, $\frac{7}{100}$, $\frac{41}{1000}$, are Decimal Fractions.

Instead, however, of writing these quantities in a fractional form, it has been agreed to put,

.3 for $\frac{3}{10}$, .07 for $\frac{7}{100}$, and .041 for $\frac{41}{1000}$;

and these substituted numbers are called Decimals; in which we observe that the number of figures in the decimal is equal to the number of cyphers in the denominator of the equivalent fraction; and where the number of significant figures is too few, a cypher or cyphers must be supplied between the full stop (.), called the decimal point, and the first significant figure.

XCIII. The reason of this method of expressing decimal fractions is apparent from the nature of the decimal notation.

For, if we take any number of ordinary arithmetic, as 57835, we see that every figure diminishes in local value as we go from the left to the right; thus the left-hand 5 represents fifty thousand, 7 but seven thousand, 8, eight hundred, 3, thirty, and the extreme 5 on the right-hand but five units; beyond this ordinary arithmetic does not proceed; but if in addition to the number just taken as

an example, we had to express 3 tenths, 4 hundredths, and 8 thousandths, it is clear that so long as we have some method of showing to the arithmetician that the division by 10 goes on beyond the unit 5 of 57835, there is no reason why 348 should not express 3 tenths, 4 hundredths, and 8 thousandths. The sole difficulty is to make a significant distinction between it and the integral part of the number; and this is effected by the decimal point (.); and thus we write for

$$57835 + \frac{3}{10} + \frac{4}{100} + \frac{8}{1000}$$

the simpler form 57835.348.

XCIV. We may also see that $\frac{3}{10} + \frac{4}{100} + \frac{8}{1000}$ may, by the definition of a decimal, be represented by .348; for, reducing the three fractions to a single fraction, with a common denominator,

$$\text{since } \frac{3}{10} = \frac{300}{1000}; \quad \frac{4}{100} = \frac{40}{1000};$$

$$\therefore \frac{3}{10} + \frac{4}{100} + \frac{8}{1000} = \frac{300 + 40 + 8}{1000} = \frac{348}{1000} = .348.$$

XCV. Hence we see that .4 stands for 4 tenths, .04 for 4 hundredths, .004 for 4 thousandths; for

$$.4 = \frac{4}{10}; \quad .04 = \frac{0}{10} + \frac{4}{100} = \frac{4}{100}, \text{ while}$$

$$.004 = \frac{0}{10} + \frac{0}{100} + \frac{4}{1000} = \frac{4}{1000}.$$

Also, conversely, any decimal may be expressed by means of a vulgar fraction.

$$\text{Thus } .237 = \frac{2}{10} + \frac{3}{100} + \frac{7}{1000} = \frac{237}{1000},$$

$$\text{and } 22.56 = 22 + \frac{5}{10} + \frac{6}{100} = 22\,\frac{56}{100}.$$

XCVI. If cyphers be added to the right-hand of the decimal, the value of the decimal is unaltered; for .5, .50, .500, &c. are the same as $\frac{5}{10}$, $\frac{50}{100}$, $\frac{500}{1000}$, &c. each of which is equal to $\frac{1}{2}$.

But if cyphers be placed to the left-hand of the decimal, between it and the decimal point, they decrease its value in a tenfold degree; thus, .5, .05, .005, &c. are 5 tenths, 5 hundredths, 5 thousandths, &c.

XCVII. Every time a decimal is multiplied by 10, the decimal point is removed one place to the right, by 100 two places, by 1000 three places, &c.; the number of cyphers in the multiplier being equal to the number of places the decimal point is moved.

Thus $53.37 \times 10 = \frac{5337}{100} \times 10 = \frac{5337}{10} = 533.7$.

And $2.014 \times 100 = \frac{2014}{1000} \times 100 = \frac{2014}{10} = 201.4$.

XCVIII. Similarly, if a decimal be divided by 10, 100, or 1000, the decimal point is removed equally to the left-hand.

Thus $15.82 \div \text{by } 10 = \frac{1582}{100} \div 10 = \frac{1582}{1000} = 1.582$.

And $11.63 \div 100 = \frac{1163}{100} \div 100 = \frac{1163}{10000} = .1163$.

Cyphers must be supplied if places be wanted, thus,

$21.8 \div 1000 = \frac{218}{10} \div 1000 = \frac{218}{10000} = .0218$.

ADDITION OF DECIMALS.

XCIX. *Rule.*—First reduce all the decimals to the same denominators, by adding, if necessary, cyphers to the right of the decimals. Then place the numbers under

each other, so that the decimal points shall be in the same vertical line. Then add as in whole numbers, placing the decimal point of the sum directly under.

Ex. (1.) Find the sum of 37.25, 18.213, 4.04, and 13.

First Method.	Usual Method.
37.250	37.25
18.213	18.213
4.040	4.04
13.000	13.
72.503	72.503

The cyphers are added to make the decimals in each number equal to the three decimal places in the second number. The reason of the rule is obviously this,—we cannot add fractions together until the denominators are all made equal. In practice the addition of cyphers to the decimals is commonly dispensed with, as in the second method.

EXAMPLES.

Find the sum of—

(2.) 17.21, 18.4, 56.1 and 3.29.
(3.) 2.003, 41.01, 94.7 and 45.601.
(4.) .002, 4.0013, 17.25 and 26.009.
(5.) 267, 54.3007, 1.82 and 825.75301.
(6.) 15.036, .00271, 1.0768 and .540545.

SUBTRACTION OF DECIMALS.

C. *Rule.*—Place the less number under the greater, the decimal points being in the same vertical line. If the number of decimal places be not the same in each, add cyphers to the deficient one, then subtract as in ordinary subtraction.

Ex. (7.) Find the difference between 815.71 and 27.415.

815.710
27.415

788.295

We may, as it is said in addition, dispense with the cypher, taking care to subtract as if it were present, thus,

815.71
27.415

788.295

EXAMPLES.

Find the difference between—

(8.) 47.813 and 23.419.
(9.) 6213.25 and 433.0067.
(10.) 747.823 and 477.3284.
(11.) 4.0014 and .40014.
(12.) 101.0101 and 1.010101.

MULTIPLICATION OF DECIMALS.

CI. *Rule.*—Multiply the factors as in whole numbers, and point off as many decimals in the product as there are in the multiplier and multiplicand together.

If the product should not contain a sufficient number of figures, the defect must be supplied by cyphers, placed to the left of the product, and between it and the decimal point.

Ex. (13.) Find the product of 4.12 and 5.7.

$$\begin{array}{r} 4.12 \\ 5.7 \\ \hline 2884 \\ 2060 \\ \hline 23.484 \end{array}$$

Three decimals in the product, since there are two in the multiplicand, and one in the multiplier.

Ex. (14.) Find the product of .102 and .7.

$$\begin{array}{r} .102 \\ .7 \\ \hline .0714 \end{array}$$

In this example the actual product of the numbers is 714, but as, by the Rule, there must be 4 decimals, a cypher is placed to the left of 7, and the decimal point placed before it.

CII. We proceed to explain the reason of the Rule.

$4.12 = 4\frac{12}{100} = \frac{412}{100}$; and $5.7 = 5\frac{7}{10} = \frac{57}{10}$.

$\therefore 4.12 \times 5.7 = \frac{412}{100} \times \frac{57}{10} = \frac{23484}{1000} =$

$23 + \frac{484}{1000} = 23.484.$

Again $.102 = \frac{102}{1000}$; and $.7 = \frac{7}{10}$.

$\therefore .102 \times .7 = \frac{102}{1000} \times \frac{7}{10} = \frac{714}{10000} = .0714.$

And the same kind of proof may be used in all cases.

EXAMPLES.

Find the product of—

(15.) 27.321 by 7.141. (16.) 41.85 by 4.185.
(17.) .02345 by .0254. (18.) 1.0101 by 101.01.
(19.) 2.0104 by .0002. (20.) .38462 by .00484.

Find the continued product of—

(21.) 1.21, 1.1, .013 and .121.
(22.) 1.001, 400.4, .01 and .003.

DIVISION OF DECIMALS.

CIII. *Rule.*—Divide as in whole numbers: but first make the divisor an integer, by removing the decimal of the dividend as many places to the right as there are decimals in the divisor. The quotient will contain as many decimals as there are in the dividend.

Ex. (23.) Divide 16.0325 by 2.75.

Remove the decimal point in both divisor and dividend two places to the right, to make the divisor a whole number, which is the same thing as multiplying both by 100, and does not affect the value of the quotient.

```
275 ) 1603.25 ( 5.83
      1375
      ----
       2282
       2200
       ----
        825
        825
        ---
```

Since the divisor is a whole number, and since the product of the divisor and quotient must, by the rule for multiplication, contain as many decimals as there are in the dividend, the number of decimals in the quotient must equal the number in the dividend.

Note.—If cyphers be added to the right of the dividend, these must be considered as decimals, in determining the position of the decimal point in the quotient.

Ex. (24.) Divide 8.2 by .0041.

$$8.2 = 8.2000.$$

Hence removing the decimal point 4 places, the divisor becomes 41, the dividend 82000.

```
41 ) 82000 ( 2000 = quotient
     82
     ---
      000
```

Ex. (25.) Divide .21 by 17; add cyphers to the right.

```
17 ) .2100000 ( .0123529
     17
     --
     40
     34
     --
      60
      51
      --
       90
       85
       --
        50
        34
        --
         160
         153
         ---
           7
```

The number of decimals in the dividend, including 5 cyphers, being 7; and there being only 6 significant figures in the quotient, a cypher is placed between them and the decimal point, in order that there may be 7 decimals in the quotient.

There is another Rule for Division commonly made use of, which renders the change of the divisor unnecessary; it is this :—

Divide as in whole numbers, and point off from the quotient as many decimals as the number in the dividend *exceeds* the number in the divisor.

Should the number of decimals in the dividend be less than those in the divisor, cyphers must be added to the right of the dividend, at least as many as will make the number of the decimals of the dividend equal to the number in the divisor.

Ex. (401.) Divide 17.084597 by .024.

```
.024 ) 17.084597 ( 711.858
       168
       ---
        28
        24
        --
         44
         24
         --
         205
         192
         ---
          139
          120
          ---
           197
           192
           ---
             5
             -
```

Here there are 6 decimals in the dividend, and 3 in the divisor, the quotient must contain 3, and the decimal point is placed between the 3d and 4th figure of the quotient.

Again, divide 14 by .7854.

Here it is necessary to add 4 cyphers to the right of

the dividend; any more will give decimal places in the quotient: we shall add two cyphers.

```
.7854 ) 14.000000 ( 17.82
        7854
       -----
        61460
        54978
       -------
         .64820
         62832
        --------
          19880
          15708
         -------
           4172
```

The division may be continued by the addition of cyphers to the dividend.

EXAMPLES.

(402.) Divide 12.78345 by 41.
(403.) Divide 237.826 by .0232.
(404.) Divide .872583 by .6734.
(405.) Divide 52.81 by .05281.
(406.) Divide 23 by .01303.
(407.) Divide 1.0157 by 47.032.

REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

CIV. As an instance, let the fraction $\frac{3}{4}$ be reduced to a decimal. Write down 3 as a dividend, and 4 for a divisor, and add cyphers to the dividend, placing a decimal point between them and 3 ; the quotient will in this case be the decimal, and the number of places in it will equal the number of cyphers made use of.

$$4 \overline{)\,3.00} \quad .75 \qquad \therefore \ \frac{3}{4} = .75.$$

For $\frac{3}{4} = \frac{30}{40} = \frac{300}{400} = \frac{1}{100} \times \frac{300}{4} = \frac{75}{100} = .75.$

Ex. (26.) Reduce $\frac{7}{128}$ to a decimal.

$$\begin{array}{r|l} 128 & 7.0000000 \ (\ .0546875 \\ & 640 \\ & \hline & 600 \\ & 512 \\ & \hline & 880 \\ & 768 \\ & \hline & 1120 \\ & 1024 \\ & \hline & 960 \\ & 896 \\ & \hline & 640 \\ & 640 \\ & \hline \end{array}$$

In this example 7 cyphers being required for the divisor, a cypher is placed to the left of the quotient, to make up the proper number of decimals.

CV. When a fraction, as $\frac{3}{4}$, can be expressed by a decimal fraction, as $\frac{75}{100}$, or by a decimal .75, the decimal is called a finite decimal; but in many cases the divisor does not terminate, but, after a certain number of times, the figures of the quotient recur; such decimals are called Recurring or Circulating Decimals, and the part which is repeated is called the Period or Repetend.

As an example, reduce $\frac{3}{7}$ to a decimal.

$$7\ )\ 3.000000$$
$$.428571428571$$

After six divisions the first dividend recurs, and consequently, the next six divisions will give the same figures as the former six, and the work can never cease. Instead of repeating the quotient, two dots are placed, one above the first, and the other above the last figure of the period; thus we write in such a case,

$$\frac{3}{7} = .\dot{4}28571\dot{4}.$$

CVI. We may reduce this fraction in another way, which will show its connexion with its decimal more completely than by the foregoing practical method.

$$\frac{3}{7} = \frac{30}{70} = \frac{1}{10}\,\frac{30}{7} = \frac{1}{10} \times \left(4 + \frac{2}{7}\right) = .4 + \frac{2}{70};$$

$$\frac{2}{70} = \frac{20}{700} = \frac{1}{100} \times \frac{20}{7} = \frac{1}{100}\left(2 + \frac{6}{7}\right) = .02 + \frac{6}{700};$$

$$\frac{6}{700} = \frac{1}{1000} \times \frac{60}{7} = \frac{1}{1000} \cdot \left(8 + \frac{4}{7}\right) = .008 + \frac{4}{7000};$$

$$\frac{4}{7000} = \frac{1}{10000} \cdot \frac{40}{7} = \frac{1}{10000} \cdot \left(5 + \frac{5}{7}\right) =$$

$$.0005 + \frac{5}{70000};$$

$$\frac{5}{70000} = \frac{1}{100000} \cdot \frac{50}{7} = \frac{1}{100000} \cdot \left(7 + \frac{1}{7}\right) =$$

$$.00007 + \frac{1}{700000};$$

$$\frac{1}{700000} = \frac{1}{1000000} \cdot \frac{10}{7} = \frac{1}{1000000} \cdot \left(1 + \frac{3}{7}\right) =$$

$$.000001 + \frac{3}{7000000};$$

whence by performing the additions:

$$\frac{3}{7} = .428571 + \frac{1}{1000000}\,\frac{3}{7}$$
$$= .428571 + .000000428571 +$$
$$= .428571428571 +$$
$$= .\dot{4}28571\dot{4}.$$

CVII. To reduce a given quantity to the decimal of another given quantity.

Reduce both quantities to the same denomination, then the fraction, which has the former quantity for its numerator, and the latter for its denominator, reduced to a decimal, will give the required result.

EXAMPLES.

(27.) What decimal of 7*s.* 6*d.* is 5*s.* 3*d.*?

$$\frac{5s.\ 3d.}{7s.\ 6d.} = \frac{63}{90} = \frac{7}{10} = .7.$$

(28.) Reduce 18*s.* $4\frac{1}{2}d.$ to the decimal of 1*l.*

The same method applies to this example; but we more commonly proceed as follows:—

4) 2.*f.*
12) 4.5
20) 18.375
.91825

We first write down the 2 farthings, and reduce them to the decimal of 1*d.*; next we reduce $4\frac{1}{2}d.$ or 4*d*. .5 to the decimal of 1*s.*; and finally reduce 18*s.* .375 to the decimal of 1*l.*

For $\frac{1}{2}d.$ is .5 of 1*d.*; and $4\frac{1}{2} = 4.5 = \frac{45}{10}$ of $1d. = \frac{45}{120}$ of 1*s.* $= .375$; and 18*s.* .375 $= \frac{18375}{1000}$ of 1*s.* $= \frac{18375}{20000}$ of 1*l.* $= .91825l.$

(29.) Reduce 2 qrs. 12 lbs. to the decimal of 1 cwt.

Here we must divide 12 by 28, or, striking out the common factor 4, divide 3 by 7.

7) 3.
4) 2.4285714̇
.6071428̇

Here the decimal may be carried on as far as we please.

(30.) Reduce 10*s.* 6*d.* and 12*s.* 5*d.* to decimals of 1*l.*

(31.) Reduce 17*s.* $10\frac{1}{2}d.$ and 4*s.* $6\frac{3}{4}d.$ to decimals of 1*l.*

(32.) Reduce 19*s.* $10\frac{3}{4}d.$ and 17*s.* $4\frac{1}{4}d.$ to decimals of 1*l.*

(33.) Reduce 2*l.* 4*s.* $8\frac{1}{2}$. to the decimal of 5*l.*

4) 2.
12) 8.5
20) 4.7083333
5) 2.23541666
.44708333̇

(34.) Reduce 1*l.* to the decimal of one guinea.

(35.) Reduce 2*s.* 3*d.* and 7*s.* 6*d.* to the decimal of a guinea.

(36.) Reduce 2 qrs. 17 lb. and 1 qr. 21 lb. to the decimal of 1 cwt.

(37.) Reduce 2 qrs. 3 nails to the decimal of a yard.

(38.) Reduce 217 yds. 2 feet 9 inches to the decimal of a mile.

(39.) Reduce 18 hours 12 minutes to the decimal of a day.

(40.) Reduce 8 wks. 5 days to the decimal of a year.

(41.) Reduce 8 oz. 14 dwts. 16 grs. to the decimal of 1 lb.

(42.) Reduce 6 bushels 2 pecks to the decimal of a quarter.

(43.) Reduce 2*l.* 12*s.* 6*d.* to the decimal of 3*l.* 10*s.*

(44.) Reduce 5 hours 48 min. 48 sec. to the decimal of a day.

(45.) Reduce 8 cwt. 2 qrs. 16 lb. to the decimal of a ton.

(46.) Reduce 2 roods 27 poles to the decimal of an acre.

CVIII. To find the value of a decimal of one denomination in terms of the known parts of its integer.

This is a repetition of the Rule in art. lxxix.

Rule.—Multiply the decimal by the parts of the next lower denomination of the integer; the integer of this product will be the first quotient: multiply the decimal part by the next lower parts of the integer, and the integral part of the product will be the second quotient; and so proceed till all the multipliers are exhausted.

EXAMPLES.

(47.) Find the value of .27435 of 1*l.*

.27435*l.*
20
5,48700*s.*
12
5,84400*d.*
4
3.376*f.*

Ans. 5*s.* 5$\frac{3}{4}$*d.* .376.

Find the value of—

(48.) .625*s.* and .83229*l.*

(49.) .6725 cwt. and .475 of a ton.

(50.) .478 of 2*l.* and .578342 of 10*s.*

(51.) .768 of a mile and .472 of a league.

(52.) .9125 of a guinea and .75 of 2*l.* 10*s.*

(53.) .4783259*l.* and 2.7834*l.*

(54.) .4758 of a week and .6854 of a day.

(55.) .4832 of an acre and 3.4562 of a yard.

RECURRING DECIMALS.

CIX. To find the vulgar fraction which is equal to the recurring decimal.

Although recurring decimals arise from fractions whose denominators are very different from each other, yet they may be all reduced to those which have either 9, 99, or 999, or numbers whose sole digit is 9, for their denominators.

Thus, since $\frac{1}{9} = .1111\ldots = \dot{1}$.

$\therefore$ $.111\ldots$ or $\dot{.1} = \frac{1}{9}$.

So also $.222\ldots$ or $\dot{.2} = \frac{2}{9}$.

$.333\ldots$ or $\dot{.3} = \frac{3}{9}$.

$.444\ldots$ or $\dot{.4} = \frac{4}{9}$.

&c.

$.888\ldots$ or $\dot{.8} = \frac{8}{9}$.

Again; since $\frac{1}{99} = .010101\ldots = .\dot{0}\dot{1}$

$$\therefore \quad .010101\ldots \text{ or } .\dot{0}\dot{1} = \frac{1}{99}.$$

$$\text{and } .070707\ldots \text{ or } .\dot{0}\dot{7} = \frac{7}{99}.$$

$$.434343\ldots \text{ or } .\dot{4}\dot{3} = \frac{43}{99}.$$

Also since $\frac{1}{999} = .001001001\ldots = .\dot{0}0\dot{1}.$

$$\therefore \quad .001001001\ldots \text{ or } .\dot{0}0\dot{1} = \frac{1}{999}.$$

$$\therefore \quad .057057\ldots \text{ or } .\dot{0}5\dot{7} = \frac{57}{999}.$$

$$\text{and } .234234\ldots \text{ or } .\dot{2}3\dot{4} = \frac{234}{999}.$$

CX. In the same manner we may proceed with higher numbers, but enough has been done to establish the following Rule.

Rule.—Every recurring decimal may be replaced by a vulgar fraction, of which the numerator is the recurring part, and the denominator a number with the digit 9 repeated as often as there are places of figures in the recurring part.

Ex. (56.) Find the vulgar fraction corresponding to $.\dot{2}\dot{7}$.

$$\text{Fraction} = \frac{27}{99} = \frac{3}{11}.$$

(57.) Find the vulgar fraction corresponding to $.\dot{4}28571\dot{4}$.

$$\text{Fraction} = \frac{428571}{999999} = \frac{47619}{111111} = \frac{15873}{37037} = \frac{5291 \times 3}{5291 \times 7} = \frac{3}{7}.$$

Ex. (58.) Find the vulgar fraction equivalent to $.\dot{0}76923\dot{}$.

$$\text{Fraction} = \frac{76923}{999999} = \frac{8547}{111111} = \frac{2849}{37037} = \frac{407}{5291} = \frac{1}{13}.$$

CXI. Sometimes the recurring decimal is composed partly of figures which do not recur; and these are succeeded by a recurring period. An example or two will show how these cases ought to be treated.

Ex. (59.) Find the fraction corresponding to .5444 or $.5\dot{4}$.

Multiply by 10 to remove the non-recurring part to the left of the decimal point, and divide by 10 also, to keep the decimal of the same value as before.

$$\text{Then } .5\dot{4} = \frac{1}{10}(5.\dot{4}) = \frac{1}{10}\left(5 + \frac{4}{9}\right) = \frac{1}{10}\,\frac{49}{9} = \frac{49}{90}$$

Ex. (60.) Find the value of $.352727 = .35\dot{2}\dot{7}$.

Here we must multiply and divide by 100.

$$\text{Then } .35\dot{2}\dot{7} = \frac{1}{100}(35.\dot{2}\dot{7}) = \frac{1}{100}\left(35 + \frac{27}{99}\right) =$$

$$\frac{1}{100} \cdot \left(35 + \frac{3}{11}\right) = \frac{388}{1100} = \frac{97}{275}.$$

A similar method applies to all cases, observing that if the non-recurring part consist of one figure, we multiply and divide by 10, if of two figures we multiply and divide by 100, if of three figures, by 1000, and so on; the cyphers of the multiplier and divisor being equal in number to the number of figures in the non-recurring part.

Ex. (61.) Find the value of $.666l.$ or $.\dot{6}l.$

$$.\dot{6} = \frac{6}{9} = \frac{2}{3}, \text{ and } .\dot{6}l. = \frac{2}{3} \text{ of } 1l. = 13s.\ 4d.$$

(62.) Find the value of $.43\dot{1}$ of a cwt.

$$.43\dot{1} = \frac{1}{10}\left(4 + \frac{31}{99}\right) = \frac{427}{990},$$

and $\frac{427}{990}$ of a cwt. $= \frac{427}{990} \times 112\text{lb.} = 48\text{lb.}\ \frac{152}{495}$.

EXAMPLES.

Find the fractions corresponding to the decimals—

(63.)	$.1313\dot{1}\dot{3}$.	(64.)	$.424\dot{2}4\dot{2}$.
(65.)	$.0909\dot{0}\dot{9}$.	(66.)	$.321321\dot{3}2\dot{1}$.
(67.)	$.\dot{1}42857\dot{}$.	(68.)	$.\dot{2}73843\dot{}$.
(69.)	$.5333\dot{}$.	(70.)	$.4\dot{7}$.
(71.)	$.51\dot{4}\dot{2}$.	(72.)	$.68\dot{4}9\dot{4}$.
(73.)	$4.032\dot{1}$.	(74.)	$17.275\dot{4}6\dot{3}$.

(75.) Find the value of $.71\dot{8}$ of 1*l.* and $.\dot{3}\dot{2}$ of a guinea.

(76.) Find the value of $.\dot{4}1\dot{3}$ of a cwt. and $.4\dot{5}$ of a lb. troy.

CONTRACTED MULTIPLICATION.

CXII. It is seldom requisite in calculations to make use of numbers with more than four or five places of decimals; the following process enables us to retain as many as may be thought necessary :—

Rule.—Write the unit's place of the multiplier under that decimal of the multiplicand whose place is the same as the number to be retained in the product, and write the remaining figures in a reverse order.

In multiplying, reject the figures of the multiplicand to the right of the first right-hand figure of the multiplier, and set down the product so that their right-hand figures may fall in a straight line directly below each other, observing to increase the first figure of every line, with what would arise from carrying 1 from 5 to 14, 2 from 15 to 24, 3 from 25 to 34, &c. from the product of the two preceding figures, when you begin to multiply, and the sum will be the product required.

Ex. (77.) Find the product of 27.654325 by 48.23, so as to retain 3 decimal places only.

Contracted.	Ordinary Method.
27.654325	27.654325
32.84	48.23
1106173	82962975
221234	55308650
5531	221234600
830	110617300
1333.768	1333.76809475

Ex. (78.) Multiply 245.378263 by 72.4385, retaining 5 places of decimals in the product.

245.378263
5834.27

1717.647841
49075653
9815130
736135
196302
12269

17774.83330

The actual product is 17774.8333043255.

CXIII. An attentive examination of these examples will show the principle on which the process is founded; which is, not to use multipliers that will produce decimals of a lower order than those required. Thus a decimal in the fourth place multiplied by a decimal in the first place must produce a decimal in the fifth place; and if four decimal places only be required, only that part of the product will be needed, which is a decimal of the fourth place; we therefore only carry the unit, if any, arising from this product. But an integer in the tens place may multiply a decimal in the fifth place, since the product removes the decimal one place to the right.

EXAMPLES.

(79.) Multiply 281.473362 by 27.45, retaining 3 decimal places.

(80.) Multiply 8764.47321 by 4.56, retaining 4 decimal places.

(81.) Multiply 72.4783256 by 42.57, retaining 5 decimal places.

(82.) Multiply .4778224 by .56732, retaining 4 decimal places.

Here use a cypher for the unit in the multiplier.

CONTRACTED DIVISION.

CXIV. To divide one decimal by another, so as to retain a specified number of decimals in the quotient.

Rule.—Take as many figures of the divisor, beginning from the left hand, as are equal to the number of integers and decimals that are to be in the quotient, and divide in the usual manner.

Consider each remainder as a new dividend, and in dividing, leave out one figure to the right hand of the divisor; remembering to carry for the figure cut off, as in the rule for contracted multiplication.

When the figures in the divisor are less in number than those in the required quotient, begin as in ordinary division, and continue the division till the number of figures in the divisor, and those remaining to be found in the quotient, are equal, after which use the contraction.

Ex. (83.) Divide 721.17562 by 2.257432, so as to retain only 3 places of decimals in the quotient.

Here the numbers of integers and decimals in the quotient will be 6.

```
2.25743 ) 721.17562 ( 319.467
          677230
          -------
           43945
           22574
           -----
           21371
           20317
           -----
            1054
             903
            ----
             151
             135
             ---
              16
              16
              --
```

EXAMPLES.

(84.) Divide 4608.29793458 by 84.31035, so as to have only 4 decimal places in the quotient.

(85.) Divide 743.71568 by 4.53246, so as to have 3 decimals in the quotient.

(86.) Divide 82.4132 by 2.682, so as to have 4 decimals in the quotient.

(87.) Divide 94.07654321 by 8.542378, so as to have 5 decimals in the quotient.

(88.) Divide 453.729843267 by 72.8413, so as to have 7 decimals in the quotient.

CHAPTER VIII.

INTEREST.

CXV. INTEREST is the sum paid to the lender by the borrower of a sum of money, for its use.

The *rate* of interest is the sum paid for a sum of money, usually 100*l.*, for a year; it is then called the rate per cent. per annum.

Thus 4 per cent. per annum, means that 4*l.* is paid for the use of 100*l.* for one year.

The sum lent is called the Principal, and the Amount is the sum of the principal and interest.

When the interest on the original sum lent, or principal, only is taken, it is called Simple Interest. But if the interest when due, at the end of any specified time, be added to the principal, and the interest of the amount be taken, it is called Compound Interest.

CXVI. To find the simple interest on a given sum for any number of years.

Rule.—Multiply the principal by the number of years, and this product by the rate per cent., and divide by 100; the quotient will be the interest.

Ex. (1.) Find the interest and amount of 647*l.* 15*s.* for 3 years, at 5 per cent. per annum.

£	s.	d.
647	15	0
		3
1943	5	0
		5
97,16	5	0
20		
3,25		
12		
3,00		

£	s.	d.
647	15	0
97	3	3
£744	18	3

Interest = 97*l.* 3*s.* 3*d.* Amount = 744*l.* 18*s.* 3*d.*

EXAMPLES.

Find the simple interest and amount of—

(2.) 826*l.* for 2 years at 3 per cent.

(3.) 1825*l.* 10*s.* for 4 years at 5 per cent.

(4.) 125*l.* 12*s.* 6*d.* for 1 year at 5 per cent.

(5.) 2000*l.* for 17 years at 4 per cent.

(6.) 625*l.* 18*s.* $4\frac{1}{2}$*d.* for 3 years at 5 per cent.

(7.) 273*l.* 3*s.* $6\frac{1}{2}$*d.* for 7 years at 3 per cent.

CXVII. When there are fractional parts in the interest, and the time consists of fractional parts of the year, we proceed as follows :—

Ex. (8.) Find the simple interest and amount of 825*l.* 12*s.* 6*d.*, for $3\frac{1}{2}$ years, at $4\frac{1}{4}$ per cent. per annum.

£	s.	d.
825	12	6
		$3\frac{1}{2}$
2476	17	6
412	16	3
2889	13	9
		$4\frac{1}{4}$
11558	15	0
722	8	$5\frac{1}{4}$
122,81	3	$5\frac{1}{4}$
20		
16,23		
12		
2,81		
4		
$\frac{3}{4}$,25		

£	s.	d.
825	12	6
122	16	$2\frac{3}{4}$
£948	8	$8\frac{3}{4}$

Interest = 122*l.* 16*s.* $2\frac{3}{4}$*d.* Amount = 948*l.* 8*s.* $8\frac{3}{4}$*d.*

EXAMPLES.

Find the simple interest and amount of—

(9.) 1835*l.* 15*s.* for $4\frac{1}{2}$ years at 2 per cent.
(10.) 273*l.* 4*s.* 6*d.* for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.
(11.) 1728*l.* 8*s.* for $5\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.
(12.) 617*l.* 8*s.* 4*d.* for $3\frac{1}{2}$ years at $3\frac{3}{4}$ per cent.
(13.) 205*l.* 15*s.* for a quarter of a year at 4 per cent.
(14.) 319*l.* 10*s.* 6*d.* for $4\frac{3}{4}$ years at $4\frac{1}{4}$ per cent.
(15.) 712*l.* 6*s.* 2*d.* for 10 years at $2\frac{3}{4}$ per cent.
(16.) 825*l.* 15*s.* 8*d.* for $3\frac{3}{4}$ years at $4\frac{3}{4}$ per cent.
(17.) 1049*l.* 18*s.* 6*d.* for $6\frac{1}{3}$ years at $3\frac{5}{8}$ per cent.
(18.) 2369*l.* 12*s.* 2*d.* for $2\frac{7}{12}$ years at $4\frac{1}{8}$ per cent.

(19.) 722*l.* 17*s.* 4*d.* for $\frac{2}{3}$ of a year at $1\frac{1}{2}$ per cent.

(20.) 4268*l.* 14*s.* 10*d.* for $\frac{1}{2}$ year at $2\frac{3}{4}$ per cent.

(21.) 925*l.* 12*s.* $6\frac{3}{4}$*d.* for $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.

(22.) 726*l.* 3*s.* $3\frac{1}{2}$*d.* for $7\frac{1}{4}$ years at $3\frac{3}{4}$ per cent.

CXVIII. When the interest is required for a number of months, multiply the principal by the number of months and the rate, and divide by 12 and by 100.

But if the time be expressed in days, find the interest for a year, and then, by the Rule of Three, or by Practice, compute the interest for the given number of days.

Ex. (23.) Find the interest of 120*l.* 12*s.* for 7 months at 3 per cent. per annum.

£	s.	d.
120	12	0
		7
844	4	0
		3
12) 2532	12	0
2,11	1	0
20		
2,21		
12		
2,52		
4		
2,08		

Interest = 2*l.* 2*s.* $2\frac{1}{2}$*d.*

Ex. (24.) Find the interest on 274*l.* from April 30th to Oct 10th, 1852, the latter day included, at 5 per cent.

£
274
5
13,70
20
14,00

13*l.* 14*s.* interest for one year.

The time is 163 days, and the interest for 365 days is 13*l.* 14*s.*

£ *s.* *d.*
∴ 365 : 163 :: 13 14 0
20
274
163
822
1644
274
365) 44662 (122*s.* = 6*l.* 2*s.*
365
816
730
862
730
132
12
1584 (4*d.*
1460
124
4
496 ($\frac{1}{4}$
365
131

Interest = 6*l.* 2*s.* 4$\frac{1}{4}$*d.* $\frac{131}{365}$.

EXAMPLES.

Find the interest on—

(25.) 250*l.* 12*s.* 6*d.* for 2 years 8 months at 4 per cent.

(26.) 1781*l.* 18*s.* for 3 years 5 months at $2\frac{1}{2}$ per cent.

(27.) 2520*l.* from March 1, 1852, to September 8, 1854, at $4\frac{1}{2}$ per cent.

(28.) 608*l.* 6*s.* 8*d.* for 350 days at 5 per cent.

(29.) 456*l.* 5*s.* for 207 days at $2\frac{3}{4}$ per cent.

(30.) 700 guineas for 4 years 87 days at $4\frac{3}{4}$ per cent.

(31.) Find the interest on an India bond for 1000*l.* for 125 days, interest being 3*l.* 12*s.* 6*d.* per cent. per annum.

(32.) Find the interest on 3 exchequer bills, one for 1000*l.* and 2 for 500*l.* each, the first being for 75 days, and the other two for 87 days each, interest being at the rate of 2*l.* 5*s.* 7*d.* per annum.

CXIX. Questions of interest are frequently worked by decimals: as instances, we will take Ex. (1.) and Ex. (8.)

Ex. (33.) Find the interest of 647*l.* 15*s.* for 3 years at 5 per cent.

£	s.		
647	15	=	647.75
			3
			1943.25
			5
			97.1625
			20
			3.2500
			12
			3.0000

Interest = 97*l.* 3*s.* 3*d.*

Ex. (34.) Find the interest of 825*l.* 12*s.* 6*d.* for $3\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.

825 12 6 = 825.625; $3\frac{1}{2} = 3.5$; $4\frac{1}{4} = 4.25$.

3.5

4128125
2476875

2889.6875
4.25

144484375
57793750
115587500

122.81171875
20

16,23437500
12

2,812500
4

3,2500

Interest = 122*l.* 16*s.* $2\frac{3}{4}$*d.* .25.

For other examples of this method, the previous ones may be made use of.

DISCOUNT.

CXX. Discount is the allowance made for the prepayment of a sum of money.

The present worth of a sum of money due at some future period, is the sum to be paid down at once, instead of at the end of the given time.

Hence, discount is the difference between the amount due at the end of the given time and its present worth. Thus, if 105*l.* be due at the end of a year, and the rate be 5*l.* per cent., the present worth will be 100*l.*, and the discount will be 5*l.* For 100*l.* paid down will, by being put to interest, amount to 105*l.* in a year; or the discount is 5*l.*, which is the sum to be deducted from 105*l.* as an allowance for the prepayment.

CXXI. *Rule.*—To find the discount, add the interest of 100*l.* for the given time to 100*l.*, and say, as this amount : : the given sum : the interest of 100*l.* : discount required; or, to find the present worth—
The amount of 100*l.* : the given sum :: 100*l.* : present worth.

But if we know the discount, we may find the present worth, by subtracting the discount from the given amount. And similarly, if we know the present worth, the discount may be found.

Ex. (35.) Find the discount on £250*l.*, due 2 years hence, at 4 per cent.

The interest of 100*l.* for 2 years at 4 per cent. = 8*l.*

£		£		£
108	:	250	:	8
		8		

108) 2000 (18*l.*
108
920
864
56
20
1120 (10*s.*
108
40
12
480 (4*d.*
432
48
4
192 (1*f.*
108
84

£	*s.*	*d.*
250	0	0
18	10	$4\frac{1}{4}\frac{7}{9}$
231	9	$7\frac{1}{2}\frac{2}{9}$

Discount = 18*l.* 10*s.* $4\frac{1}{4}$*d.* $\frac{7}{9}$.
Present worth = 231*l.* 9*s.* $7\frac{1}{2}$*d.* $\frac{2}{9}$.

Ex. (36.) Find the present worth of the same sum.

£ £ £
108 : 250 : 100
100

108) 25000 (231*l.*
216

340
324

160
108

52
20

1040 (9*s.*
972

68
12

816 (7*d.*
756

60
4

240 (2*f.*
216

24

Present worth = 231*l.* 9*s.* $7\frac{1}{2}$*d.* $\frac{2}{3}$.

EXAMPLES.

(37.) Find the discount on 525*l.* due 3 years hence, discounting at 4 per cent.

(38.) Find the present worth of 150*l.* due $1\frac{1}{4}$ year hence, discounting at 3 per cent.

(39.) Find the discount on 275*l.* 10*s.* due 3 years hence, at 4 per cent.

(40.) Find the discount on 875*l.* 6*s.* 8*d.* for $1\frac{1}{2}$ year, at $4\frac{1}{2}$ per cent.

(41.) Find the present worth of a bill for 690*l.* 12*s.* due 9 months hence, at $2\frac{1}{2}$ per cent.

(42.) Find the discount on 1250*l.* due 75 days hence, discounting at $3\frac{1}{2}$ per cent.

CXXII. The method given in the preceding article ascertains correctly the discount and present worth; but in business interest is always charged instead of discount.

Thus, when a merchant presents a bill to a banker, the latter calculates the interest on the sum to be paid, for the time that the bill has to run; to which time he also adds three days of grace, (the time allowed for payment after the date mentioned in the bill,) and this sum he calls discount.

According to this, the discount of 105*l.* due a year hence, at 5 per cent., would be 5*l.* 5*s.* instead of 5*l.*, which is manifestly unjust, since the banker will, at the end of the year, receive 105*l.* for the loan of 99*l.* 15*s.*, while the merchant will receive 99*l.* 15*s.* and its interest, which is 4*l.* 19*s.* 9*d.*, or 104*l.* 14*s.* 9*d.* instead of 105*l.*, which he ought to receive.

The following are examples of Commercial Discount—

July 1, 1852. A merchant discounted the following bills at 4 per cent.

(43.)	105*l.*	12*s.*	6*d.*	due Aug. 10.
(44.)	1250*l.*	10*s.*	0*d.*	due Aug. 20.
(45.)	350*l.*	0*s.*	0*d.*	due Sept. 8.
(46.)	275*l.*	18*s.*	4*d.*	due Sept. 20.
(47.)	496*l.*	12*s.*	4*d.*	due Oct. 10.
(48.)	4256*l.*	8*s.*	8*d.*	due Oct. 21.

Find the amount received, three days' grace being allowed.

COMPOUND INTEREST.

CXXIII. In Compound Interest, the interest when due is added to the principal, and itself becomes principal, and the interest of it is taken in the same way as the interest of the original principal.

EXAMPLES.

(49.) Find the compound interest on 500*l.* for 4 years, at 5 per cent.

Instead of multiplying by 5, and dividing by 100, we may only divide by 20.

	£				
20)	500				
	25				
20)	525		amount at beginning of		2d year.
	26	5			
20)	551	5	ditto		3d year.
	27	11	3		
20)	578	16	3	ditto	4th year.
	28	18	$9\frac{3}{4}$		
	607	15	$0\frac{3}{4}$	at end of	4th year.
	500	0	0		
Interest =	107	15	$0\frac{3}{4}$		

The simple interest for the same time would be 100*l.*

(50.) Find the compound interest on 2,000*l.* for 6 years, at 4 per cent.

(51.) Find the compound interest on 1,000,000*l.*, for 10 years, at 5 per cent.

(52.) Find the compound interest on 820*l.* 10*s.* for $2\frac{1}{2}$ years, at 3 per cent.

COMMISSION, INSURANCE, BROKERAGE, STOCKS.

CXXIV. Commission is the allowance of a sum of money, generally so much per cent., but often so much in the pound sterling, for the transaction of the business of a distant employer.

Insurance is a sum paid to make good a loss which may happen through fire, shipwreck, or storm.

Brokerage is an allowance similar to commission, paid to persons authorized to buy or sell certain articles.

Stocks are debts due by various governments or companies, and on which a fixed interest is paid.

Thus the English stocks are 3 per cents., 3¼ per cents. &c., and are called consols, or consolidated, since various loans granted to the Government at different times, and at different rates of interest, were, by act of Parliament, consolidated and formed into one stock.

All questions of the above-mentioned classes may be worked like those in simple interest, but the time forms no part of the work.

Ex. (53.) What is the commission on 484*l.* 12*s.* 6*d.* at 2½ per cent. ?

£	s.	d.	
484	12	6	
		2½	
969	5	0	
242	6	3	
12,11	11	3	
20			
2,31			
12			
3,75			
4			
3,00			12*l.* 2*s.* 3¾*d.*

(54.) Find the insurance on 750*l.* at 2 guineas per cent.

£	
750	
$2\frac{1}{10}$	
1500	
75	
15,75	
20	
15,00	15*l.* 15*s.*

(55.) Find the brokerage on 1242*l.* at 2*s.* 6*d.* per cent.

£	
1242	
$\frac{1}{8}$	
1,55 ., 5*s.*	
20	
11,05	
12	
,60	
4	
2,40	1*l.* 11*s.* $0\frac{1}{2}$*d.* .4

(56.) What is the purchase of 2500*l.* 3 per cent. stock at $99\frac{3}{8}$ per cent. ?

25	2500 at 100 per cent.
3	25
8) 75	∴ 2475 at 99 per cent.
£9 7 6	9 7 6 for $\frac{3}{8}$.
	is £2484 7 6

We might have multiplied by $99\frac{3}{8}$, and then divide by 100.

Observe that in the purchase or sale of stock, we must add or subtract $\frac{1}{8}$ per cent. for brokerage.

EXAMPLES.

(57.) Find the commission on 202*l.* 1*s.* 8*d.* at $2\frac{1}{2}$ per cent.

(58.) What is the commission on 3720*l.* at $3\frac{1}{8}$ per cent.?

(59.) What is the allowance on 1059*l.* 16*s.* 10*d.* at $2\frac{1}{4}$ per cent.?

(60.) What is the commission on 2850*l.* at $3\frac{1}{8}$ per cent.?

(61.) A ship in harbour is insured for 3,000*l.* at 3*s.* per cent., what is the amount of the insurance?

(62.) If I insure my house for 2,500*l.* at 5*s.* 3*d.* per cent., what shall I pay?

(63.) Bank stock being 256 per cent., how much can I buy for 2,000*l.*?

(64.) What income will arise from investing 1,009*l.* 10*s.* in the 3 per cents. at $84\frac{1}{8}$?

Since $84\frac{1}{8}$, or 84*l.* 2*s.* 6*d.* receives 3*l.*

	£	s.	d.		£	s.		£
∴	84	2	6	:	1009	10	::	3
	20				20			
	1682				20190			
	12				12			
	20190				242280			

Instead of multiplying the second term by 3, divide the first by 3.

Then 673,0) 242280 (36*l.* answer.

2019

4038

4038

(65.) If 3,000*l.* be invested in the 3 per cents., at 99 $\frac{3}{8}$ per cent.; find the interest, brokerage $\frac{1}{8}$ per cent.

(66.) A person lent 1,000*l.* on mortgage at 4 per cent. and bought 1,000*l.* in the 3 per cents. at 74 $\frac{7}{8}$; what amount of interest did he receive on both sums?

(67.) How much railway stock at 125*l.* 10*s.* per cent. can be purchased for 3,500*l.*?

(68.) And if the dividend be 5*l.* on each 100*l.* stock, what will be the dividend on the investment?

(69.) A person transfers 1,000*l.* in the 3 per cents. at 92 into the 3$\frac{1}{2}$ per cents. at 96, what does he gain or lose?

(70.) A person invests 2,000*l.* in the 3 per cents. when they are at 91 per cent., and gains 100*l.* when he sells out; what was the selling price?

MISCELLANEOUS.

(71.) What principal put out to interest for 6 years at 4 per cent. would amount to 600*l.*, reckoning simple interest?

(72.) Find the compound interest on 32*l.* 12*s.* 10$\frac{1}{2}$*d.* for 2 years 7 months and 73 days at 3$\frac{1}{4}$ per cent.

(73.) What is the present value of 1,000,000*l.* due 1,000 years hence, at 4 per cent.?

(74.) A man buys an estate, which lets for 450*l.* a year, at 28 years' purchase: how much stock in the 3 per cents. at 96 must he sell out to pay for the estate; and what is his annual gain?

(75.) What is the price of an annuity of 500*l.* a year at 12.258 years' purchase; and what is the rate per cent.?

(76.) A person borrowed 1,000*l.* at 4$\frac{1}{2}$ per cent., on Jan. 1; he is to repay 300*l.* on Lady Day, 250*l.* at Midsummer, 250*l.* on Oct. 1, and the rest with the whole interest on Dec. 26; how much will the last payment be?

(77.) A bill for 1,050*l.*, dated March 1, at 3 months' date, was discounted April 16; what was received for the bill?

(78.) If the 3 per cent. stock pay 3$\frac{1}{2}$ per cent. to the invester, what is the price of the stock, and what ought to be the corresponding price of the 3$\frac{1}{2}$ per cent. stock?

CHAPTER IX.

EQUATION OF PAYMENTS.

CXXV. By this Rule we find the time at which two or more debts, due at different times, may be paid without loss either to the debtor or creditor.

Rule.—Multiply each debt by the time at which it is due, then divide the sum of the products by the sum of the debts, and the quotient will be the equated time.

Ex. (1.) *A* owes 500*l.*, 200*l.* is due at the end of 9 months, and 300*l.* at the end of 4 months; when should the whole be paid at once?

$$\begin{array}{rcl} 200 \times 9 & = & 1800 \\ 300 \times 4 & = & 1200 \end{array}$$

$$500 \overline{)\, 3000} \; (\; 6 \text{ months.}$$
$$3000$$

The truth of the Rule may be shown from this example:—

200*l.* is paid 3 months *before*, 300*l.* is paid 2 months *after* it is due; and as the interest of 200*l.* for 3 months is the same as that of 300*l.* for 2 months, the interest *A* loses by prepayment is exactly equal to that gained by postpayment. The method is a simple example of algebra.

EXAMPLES.

(2.) *A* owes 100*l.* in 8 months, and 200*l.* in $1\frac{1}{2}$ year; find the equated time.

(3.) If 150*l.* be due in 2 months, 300*l.* in 4 months, 600*l.* in 6 months, when should they be all paid at once?

(4.) If 66*l.* 13*s.* 4*d.* be due in 3 months, 33*l.* 6*s.* 8*d.* in 4 months, 200*l.* in 5 months, 120*l.* 13*s.* 4*d.* in 6 months, when should the whole be paid?

(5.) A debt of 2,400*l.* is to be paid by monthly payments of 200*l.*, when should it be paid at once?

(6.) *A* owes *B* 400*l.* to be paid in 6 months; in $1\frac{1}{2}$ month he pays 120*l.*, and in $4\frac{1}{2}$ months more he pays 140*l.*, when should he pay the rest?

(7.) If 100*l.* is due now, and 100*l.* at the end of every month for 12 months, when should the whole be paid?

(8.) A debt is to be discharged by paying one-half of it now, and one-tenth of the remainder every month; when should the whole be paid at once?

PROFIT AND LOSS.

CXXVI. By this Rule is ascertained the gain or loss in the sale of articles. In general, tradesmen speak of the gain or loss per cent., or what they would lose or gain if they sold articles to the amount of 100*l.* Questions of this kind are commonly done by Proportion. But if the absolute loss or gain in any particular transaction be required, the rules of Compound Multiplication or Division will be sufficient.

Ex. (9.) Having sold an article for 1*s.* $1\frac{1}{2}$*d.* which cost 1*s.*, what is the gain per cent.?

Here $1\frac{1}{2}d.$ is the gain on one article, and $1s.$: $1\frac{1}{2}d.$ :: $100l.$

$$24 : 3 :: 100$$

$$3$$

$$24) 300 (12l.$$
$$24$$
$$60$$
$$48$$
$$12$$
$$20$$
$$240 (10s.$$
$$240$$

$12l.$ $10s.$ or $12\frac{1}{2}$ per cent.

Or thus, since $1\frac{1}{2}d.$ is $\frac{1}{8}$ of a shilling, $8 : 1 :: 100l.$

$$\therefore \text{ gain } = \frac{100}{8} = 12\tfrac{4}{8} = 12\tfrac{1}{2}l.$$

Ex. (10.) Tea is sold at $8s.$ $3d.$ per lb., at a gain of 10 per cent., what was the cost price?

Here what was sold for $110l.$ cost $100l.$

$$\therefore \quad \underset{£}{110} : \underset{£}{100} :: \underset{s.}{8}\ \underset{d.}{3}$$

$$12$$
$$99$$
$$100$$
$$11{,}0) 99{,}00$$
$$12) 90$$
$$7s.\ 6d.$$

Answer, $7s.$ $6d.$

Ex. (11.) Wheat cost a merchant $2l.$ $5s.$ a quarter, and he sells it at a loss of 6 per cent.; what was the selling price per quarter?

Here, what cost 100*l.* is sold for 94*l.*

£ £ £ *s.* *d.*
100 : 94 :: 2 5 0

20

45
94

180
405

100) 423,0

s. 42,3
12

10) 36

d. 3 6
4

f. 2,4

Answer, 2*l.* 2*s.* $3\frac{1}{2}$*d.* .4.

Ex. (12.) Having sold cloth for 2*s.* 8*d.* per yard, I lost 4 per cent., whereas I intended to have gained 8 per cent.; what ought to have been the selling price?

First, to obtain the cost price—

s. *d.*
96 : 100 :: 2 8

12

32
100

96) 3200 (33*d.*
288

320
288

32

Cost price is 33 $\frac{8}{96}$*d.* or 33 $\frac{1}{3}$*d.*

Next, to obtain the intended selling price—

$$100 : 108 :: \overset{d.}{33\tfrac{1}{3}},$$

$$\text{or, } 100 : 108 :: \frac{100}{3}.$$

$$\text{Selling price} = \frac{108 \times \frac{100}{3}}{100} = 36d. = 3s.$$

EXAMPLES.

(13.) If cloth cost 8*s.* 4*d.* per yard, at what price must it be sold to gain 12 per cent.?

(14.) Tea which cost 4*s.* per lb. is sold for 4*s.* 6*d.*; what is the gain per cent.?

(15.) At $1\frac{1}{4}d.$ in the shilling profit, what is the gain per cent.?

(16.) Butter is bought for 1*s.* per lb., and is sold for 1*s.* 1*d.*; what is the gain per cent.?

(17.) Butter is bought for 1*s.* 1*d.* per lb., and is sold for 1*s.*; what is the loss per cent.?

(18.) If by selling wheat at 42*s.* per quarter I gain 5 per cent., what must be the price to gain 6 per cent.?

(19.) A grocer buys sugar at 50*s.* per cwt., and retails it at $5\frac{3}{4}d.$ per lb.; what is his gain per cent.?

(20.) By selling iron at 2*l.* 5*s.* per ton there is a loss of 10 per cent., but a gain of 6 per cent. was anticipated; for how much was the iron sold per ton below the anticipated price?

(21.) Wine was sold at 35*s.* per dozen, and $12\frac{1}{2}$ per cent. was lost; what was the prime cost?

(22.) An article cost 6*s.* 8*d.*; at what price must it be sold to gain 15 per cent.?

(23.) If sugar be bought at 2*l.* 14*s.* 6*d.* per cwt., what must it be sold for per lb. to gain 5 per cent.?

(24.) An edition of 4,000 copies of a work, selling price 5*s.*, is published, discount to the trade is 27 per cent., advertising and other expenses 7 per cent., author's profit 135*l.*; how much did the publisher receive for the cost of printing and for his own profit?

(25.) A chest of tea weighing 87½lbs. cost 19*l.* 1*s.* 3*d.*; at what rate per lb. must it be sold to gain 5*l.* by the whole, and what is the gain per cent.?

(26.) A grocer mixes 80lbs. of tea at 3*s.* 8*d.*, 60lbs. at 4*s.*, 40lbs. at 4*s.* 3*d.*, and 20lbs. at 4*s.* 6*d.* together; how must he sell the mixed tea to gain 3*l.* by it, and what is that per cent.?

(27.) A market woman sells eggs 10 for a shilling, having bought them 14 for a shilling; what is the gain per cent.?

(28.) If 10 per cent. be gained by selling tea at 4*s.* 6*d.* per lb., what must be the selling price to gain 12 per cent.; and what is the prime cost per lb.?

BARTER.

CXXVII. Barter is the exchanging of one kind of goods for another of equal value.

There is no general rule by which questions of this kind are to be worked out. They may be done by the rule of Proportion, but it will be best to leave the method to the skill of the arithmetician, who will for the most part find out that which is best suited to the particular example.

Ex. (29.) How much sugar at 8*d.* per lb. must be given for 20 cwt. of tobacco at 3*l.* per cwt.?

The value of the tobacco is 60*l.* or 14400*d.*

$$\therefore \text{ lbs. of sugar} = \frac{14400}{8} = 1800 = 16\text{cwt. } 8\text{lbs.}$$

Or thus,

£
3
20
60
20
1200
12
8) 14400
112) 1800 (16 cwt.
112
680
672
8 lbs.

EXAMPLES.

(30.) Exchanged 516 yards of cloth at 11*s.* 4*d.* per yard, for 688 pairs of silk stockings, what was the price of each pair?

Value of cloth = 11*s.* 4*d.* × 516 = 292*l.* 8*s.* = 5848*s.*

Price of a pair = $\frac{5848}{688}$ = 8*s.* $\frac{344}{688}$ = 8$\frac{1}{2}$*s.* = 8*s.* 6*d.*

(31.) How many yards of cloth, at 23*s.* 8*d.* per yard, are worth 18 cwt. 2 qrs. of sugar at 56*s.* per cwt.?

(32.) A farmer sells 20 quarters of wheat at 46*s.* for 8 quarters of barley at 26*s.*, and for oats at 21*s.*; how much oats did he buy?

(33.) How much cheese at 44*s.* per cwt. must *A* give *B* for 20*l.* in cash and 44 cwt. 16 lbs. of raisins at 6$\frac{1}{2}$*d.* per lb.?

(34.) How much tea at 5*s.* per lb. must be exchanged for 4 cwt. 2 qrs. of chocolate at 2*s.* per lb.?

(35.) *A* barters 1,000 yards of calico at $4\frac{3}{4}d.$ per yard for sheeting at $1s.$ $2\frac{1}{4}d.$ per yard; how many yards of sheeting ought *A* to receive?

(36.) *A* charges *B* in barter $36s.$ a dozen for wine, which he sells at $31s.$ $6d.$ a dozen for ready money; what must *B* charge *A* per gallon for brandy, which he sells at $25s.$ a dozen ready money?

(37.) How much printed stuff at $2s.$ $9\frac{1}{2}d.$ per yard is worth 17 casks of butter at $58s.$ per cask, and $8l.$ $12s.$ $6d.$ in money?

(38.) What is sugar a lb. when I receive 14 cwt. $18\frac{2}{3}$lbs. for 17 cwt. of tobacco at $3l.$ $10s.$ per cwt.?

(39.) With 3 cwt. 3 qrs. of sugar at $3l.$ $4s.$ a cwt. I purchase 16 yards $2\frac{6}{9}$ qrs. of cloth; what is the price per yard?

(40.) How much coffee at $7l.$ $9s.$ $6\frac{3}{4}d.$ per cwt. should I give for 897 cwt. 1 qr. 14 lbs. of sugar at $6\frac{3}{4}d.$ per lb.?

FELLOWSHIP, OR PARTNERSHIP.

CXXVIII. This is a rule by which merchants trading in partnership with a joint stock may ascertain each person's share of the gain or loss, in proportion to his share in the joint stock.

Fellowship is simple or compound. Simple Fellowship is when the different shares are employed for the same equal time. Compound Fellowship is when the times the shares are employed are unequal.

SINGLE FELLOWSHIP.

Rule.—As the whole stock : the whole gain or loss :: each man's stock : his share of the gain or loss.

Proof.—Add together all the shares of the gain and loss, and their sum ought to be equal to the whole gain or loss.

Ex. (41.) *A* and *B* join in a trade and gain 160*l.*; *A* advanced 300*l.*, *B* advanced 700*l.*; what should each receive?

Total capital = 300 + 700 = 1000.

1000 : 160 : 300
100 : 16 : 300
1 : 16 : 3
3

£48 = *A's* share.

1000 : 160 :: 700
1 : 16 :: 7
7

£112 = *B's* share.

Proof 48*l.* + 112*l.* = 160*l.*

EXAMPLES.

(42.) Divide 360*l.* between 3 persons that their shares may be to each other as the numbers 1, 2, 3.

Since 1 + 2 + 3 = 6 : one part will be $\frac{1}{6}$, another will be $\frac{2}{6}$, and the third $\frac{3}{6}$ of the whole.

Now $\frac{1}{6}$ of 360 = 60 : $\frac{2}{6}$ of 360*l.* = 120 : and $\frac{3}{6}$ of 360 = 180.

The parts are 60*l.*, 120*l.* and 180*l.*

(43.) A person ordered 1,000*l.* to be divided among his three sons, so that *A* might have $\frac{1}{3}$ part, *B* $\frac{1}{4}$, and *C* $\frac{1}{5}$; what is the just share of each?

[This question has been given by old arithmeticians, and means that the parts are to be in that proportion.]

Reduce $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, to the fractions with the same denominator, they become $\frac{20}{60}$, $\frac{15}{60}$, and $\frac{12}{60}$.

Hence the parts are as the numbers 20, 15, and 12, and $20 + 15 + 12 = 47$, which may represent the whole stock.

		£		£
47	: 20 ::	1000	: A's share =	$425\frac{25}{47}$
47	: 15 ::	1000	: B's share =	$319\frac{7}{47}$
47	: 12 ::	1000	: C's share =	$255\frac{15}{47}$
			Proof . .	£1000

(44.) The joint stock of A and B is 224*l.*, A contributes 96*l.* and B the rest; they gain 28*l.*; what is the gain of each?

(45.) A and B lose 1,000*l.* in a speculation, the capital of A is 3,500*l.*, and of B is 5,000*l.*; what loss will each sustain?

(46.) Four persons, A, B, C, D, enter into partnership; A pays 180*l.*, B 240*l.*, C 350*l.*, and D 430*l.*; they gain 360*l.*; what is each person's share of the gain?

(47.) Water, brandy, and wine, are mixed together in the proportion of the numbers 1, 4, and 15, the mixture contains 1,000 gallons; what are the respective quantities of water, brandy, and wine?

(48.) The estate of a bankrupt produces 675*l.* 15*s.*; he owes A 275*l.* 14*s.*, B 304*l.* 7*s.*, C 152*l.*, D 104*l.* 6*s.*; how much does each creditor receive?

(49.) A ship worth 1,000*l.* is entirely lost. A had $\frac{1}{3}$, B had $\frac{1}{4}$, C had the rest, there is a policy of insurance for 350*l.*; find the respective losses of A, B, and C.

(50.) A, B, C, purchase a house for 1,600*l.*, of which A paid 240*l.*, B 960*l.*, and C the rest; they sell it for 1,760*l.*; what will each person receive?

(51.) A and B rent a field for 25*l.*; A puts in 27 oxen,

and *B* puts in 180 sheep; if each ox eat as much as 10 sheep, what should each pay of the rent?

(52.) *A* and *B* rent a field for 18*l*.; *A* puts in 14 horses, and *B* 23 cows; if 2 horses eat as much as 3 cows, what should each pay?

DOUBLE FELLOWSHIP.

CXXIX. *Rule.*—Multiply each man's stock by the time it continues in trade, then the sum of all the products : the whole gain or loss :: each man's product : his share of the gain or loss.

Ex. (53.) *A* and *B* trade together; *A* invests 400*l*. for 3 months, *B* 200*l*. for 4 months; they gain 100*l*.; what should each receive?

$$\begin{aligned} 400 \times 3 &= 1{,}200 \\ 200 \times 4 &= \quad 800 \\ \hline & \quad 2{,}000 \end{aligned}$$

2,000 : 100 :: 1,200 : *A*'s gain = 60*l*.
2,000 : 100 :: 800 : *B*'s gain = 40*l*.

The reason of the rule is this:—That *A*'s capital of 400*l*. for 3 months is the same as having 1,200*l*. for 1 month, and *B*'s capital of 200*l*. for 4 months is the same as having a capital of 800*l*. for 1 month; if therefore we consider *A* and *B* to have a capital of 1,200*l*. and 800*l*. for the same time, we must find the proportion of their gains as above, which is then the same as the Rule for Single Fellowship.

EXAMPLES.

(54.) *A* and *B* rent a field for 82*l*.; *A* puts in 64 horses for 25 days, *B* 56 horses for 30 days; what should each pay of the rent?

$$\begin{aligned} 64 \times 25 &= 1{,}600 \\ 56 \times 30 &= 1{,}680 \\ \hline & \quad 3{,}280 \end{aligned}$$

3,280 : 1,600 :: 82*l.* : *A*'s share.
41 : 20 :: 82*l.* : *A*'s share.
1 : 20 :: 2*l.* : *A*'s share.

A's share = 40*l.*

3,280 : 1,680 :: 82*l.* : *B*'s share.
41 : 21 :: 82*l.* : *B*'s share.
1 : 21 :: 2*l.* : *B*'s share.

B's share = 42*l.*

(55.) *A* and *B* enter into partnership; *A* advances 150*l.* for 2 years, and *B* 120*l.* for $3\frac{1}{2}$ years; they gain 60*l.*; what does each gain?

(56.) *A* and *B* rent a field for 27*l.*; *A* puts in 15 oxen for 10 days, *B* 21 oxen for 7 days; what rent should each pay?

(57.) *A*, *B*, and *C*, enter into trade; *A* advances 100*l.* for 9 months, *B* advances 150*l.* for 7 months, *C* 80*l.* for 10 months; they gain 55*l.*; what is the gain of each?

(58.) *A* and *B* rent a field for 40*l.*, *A* puts in 8 horses for 7 months, *B* puts in 24 oxen for 8 months; supposing 2 horses to eat as much as 3 oxen, what is the proportion of the rent that each should pay?

(59.) *A*, *B*, *C*, hired a piece of land for 60*l.* 10*s.*; *A* put in 5 sheep for $4\frac{1}{2}$ months, *B* put in 8 for 5 months, *C* put in 9 for $6\frac{1}{2}$ months; how much must each pay of the rent?

(60.) *A* and *B* enter into partnership for 18 months; *A* put in 2,000*l.*, and 8 months after added 1,000*l.* more, *B* put in 5,500*l.*, and at the end of 4 months took out 1,400*l.*; at the end of the time they have gained 1,315*l.*; what is the gain of each?

CHAPTER X.

DUODECIMALS.

CXXX. DUODECIMALS is a rule by which the areas of floors and walls, and the solidities, or contents of rooms and buildings, contained by perpendicular walls on horizontal floors, may be found. It is used to find the work done by workmen and artificers, such as bricklayers, masons, carpenters, painters, and glaziers.

It is called Duodecimals, since, the unit being a foot, the inch is $\frac{1}{12}$th of a foot, and hence, in multiplying we carry 1 for every 12, instead of 1 for 10 as in ordinary decimals.

Rule 1.—Set down the two quantities to be multiplied directly below each other, so that feet may stand under feet, inches under inches, &c.

2. Multiply each term of the multiplicand, by the feet in the multiplier, beginning at the lowest, and place the result under the corresponding term of the multiplicand, carrying 1 for every 12.

3. Next multiply by the inches of the multiplier, and set the result of each term, one place to the right hand of the corresponding place in the multiplicand, and so on for any other multiplier.

Add the products for the total results.

Ex. (1.) Find the product of 7 feet 5 inches, by 9 feet 7 inches.

ft.	in.	
7	5	
9	7	
66	9	
4	3	11
71	0	11

Answer, 71 feet, 11 inches.

We will now do the question by fractions, and then the reason will appear why the third number 11 is called inches.

$$7 \text{ feet } 5 \text{ inches} = 7 + \frac{5}{12};$$

$$9 \text{ feet } 7 \text{ inches} = 9 + \frac{7}{12}.$$

$$7 + \frac{5}{12}$$

$$9 + \frac{7}{12}$$

$$\times \text{ by } 9 \qquad 63 + \frac{45}{12}$$

$$\times \text{ by } \frac{7}{12} \qquad + \frac{49}{12} + \frac{35}{144}$$

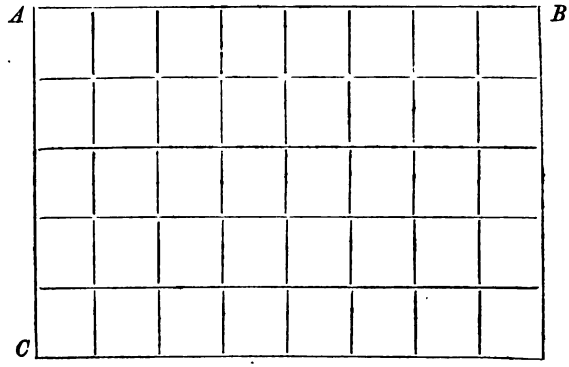
$$63 + \frac{94}{12} + \frac{35}{144}$$

But $\frac{35}{144} = \frac{24 + 11}{144} = \frac{2}{12} + \frac{11}{144}$.

$\therefore$ Product $= 63 + \frac{96}{12} + \frac{11}{144} = 71 + \frac{11}{144}$.

And since there are 144 inches in a square foot, $\frac{11}{144}$ of such a foot is 11 inches.

CXXXI. That the area of a figure contains as many square feet as the product of the numbers of the feet in length in two adjacent sides, may be seen by the accompanying figure:—



A B is supposed to be 8 feet in length, *A C* is supposed to be 5 feet in length, and the figure will be found to contain 8 × 5, or 40 divisions, each a foot square.

Ex. (2.) The length of a room is 25 feet 6 inches, its breadth is 14 feet 10 inches; find how many yards, feet, and inches are in the floor.

	ft.	in.	
	25	6	
	14	10	
	357	0	
	21	3	0
feet	378	3	

9) 378

42 yds. and $3 = \frac{3}{12}$ of a foot = 36 inches.

Answer, 42 yards, 0 feet, 36 inches.

CXXXII. In solid or cubic measure we multiply the height, length, and breadth together.

Ex. (3.)—Find the solidity of a stone 13 feet 8 inches long, 7 feet 9 inches broad, and 3 feet 11 inches thick.

ft.	in.		
13	8		
7	9		
95	8		
10	3		
105	11		
3	11		
317	9		
97	1	1	
414	10	1	

Answer, 414 feet 10′ 1″.

The second number in this result is usually marked with a single mark, the third number, called a second, with two marks, and if there were another it would be called a third, and distinguished by three marks.

The 10′ are not to be called inches, for it is $\frac{10}{12}$ths of a foot, and a solid foot contains 1728 cubic inches, and 1″ is $\frac{1}{144}$th of a foot, or 12 cubic inches; hence, since, $\frac{10}{12}$ths $= \frac{1440}{1728}$, the total result will be,

414 feet, 1440 inches and 12 inches;
or, 414 feet, 1452 cubic inches.

EXAMPLES.

Multiply—

(4.) 14 feet 7 inches by 3 feet 11 inches.
(5.) 10 feet 8 inches by 11 feet 4 inches.
(6.) 11 feet 5 inches by 7 feet 8 inches.

(7.) 18 feet 10 inches by 4 feet 6 inches.

(8.) 10 feet 11 inches by 12 feet 7 inches.

(9.) 2 feet 3 inches 4″ by 4 feet 8 inches.

(10.) 17 feet 4 inches 6″ by 9 feet 5 inches.

(11.) 23 feet 10 inches 8″ by 5 feet 7 inches 10″.

(12.) 42 feet 11 inches 7″ by 17 feet 8 inches 9″.

(13.) Find the solid feet in a cube, 11 feet 2 inches every way.

(14.) Find the solidity of a stone, 18 feet 4 inches long, 17 feet 4 inches broad, and 20 feet 10 inches high.

(15.) Find the content of a piece of timber 32 feet 6 inches long, 2 feet 5 inches broad, and 14 inches thick.

(16.) The dimensions of a cistern are, depth 8 feet 6, area of the bottom a square of 7 feet 6; how many imperial gallons will it hold?

CXXXIII. The price of work of this kind may be found by Practice; the price of a yard or a foot being known.

Thus, if in Example 1, we wish to find the expense of glazing a window of the given dimensions at 1*s.* 8*d.* per foot.

1st. $1s.\ 8d. \times 71 = 5l.\ 18s.\ 4.$

2d. Cost of $11'' = 20d. \times \frac{11}{144} = \frac{55}{36} = 1d. \frac{19}{36} = 1\frac{1}{2}\frac{1}{18}.$

Ans. $5l.\ 18s.\ 5\frac{1}{2}d.\ \frac{1}{18}.$

EXAMPLES.

(17.) Find the price of a marble slab, length 7 feet 5 inches, and breadth 3 feet 2 inches, at 5*s.* a foot.

(18.) What will the paving of a court-yard come to at $5\frac{1}{2}d.$ per yard, the length being 45 feet 10 inches, and the breadth 32 feet 8 inches?

(19.) A room 25 feet 8 inches long, and 16 feet 9 inches wide, is to be ceiled; what will be the cost at 2*s.* $10\frac{1}{2}d.$ per yard?

(20.) There is a house with 3 tiers of windows 5 in a tier, the breadth of all the windows is the same, 3 feet 10 inches; but the height of the lower tier is 7 feet 10 inches, of the second 7 feet 4 inches, of the third 5 feet 8 inches; what will the glazing come to at 1*s.* 4*d.* per foot?

SQUARES AND CUBES OF NUMBERS. SQUARE ROOTS AND CUBE ROOTS.

CXXXIV. The product of a number by itself is called its square.

The product of a number by itself twice repeated is called its cube.

Thus $4 \times 4 = 16$, or 16 is the square of 4.

And $4 \times 4 \times 4 = 64$, or 64 is the cube of 4.*

CXXXV. The square root of a number is that factor which, multiplied by itself, will produce the number.

The cube root of a number is that factor which, multiplied by itself twice, will produce the given number.

Hence the square root of 16 is 4.

And the cube root of 64 is 4.

Since $4 \times 4 = 16$; and $4 \times 4 \times 4 = 64$.

We shall now give methods by which the square and cube roots of any numbers may be found either exactly or sufficiently near for practical purposes; but it will be advisable to commit to memory the squares and cubes of the first twelve numbers.

CXXXVI. The following table contains the squares and cubes, and the square and cube roots of the first twelve natural numbers.

* The square of a number is often expressed by placing a $(^2)$ above the number, and the cube by a $(^3)$; thus 4^2 is the square, and 4^3 is the cube of 4.

Numbers.	Squares.	Cubes.	Square Roots.	Cube Roots.
1	1	1	1.	1.
2	4	8	1.41421	1.25992
3	9	27	1.73205	1.44225
4	16	64	2.	1.58740
5	25	125	2.23606	1.70997
6	36	216	2.44948	1.81712
7	49	343	2.64575	1.91293
8	64	512	2.82842	2.
9	81	729	3.	2.08008
10	100	1000	3.16227	2.15443
11	121	1331	3.31662	2.22398
12	144	1728	3.46410	2.28942

CXXXVII. Since the square of 1 is 1; of 10 is 100; of 100 is 10,000; and since 10 is the least number with two digits; and 100 is the least number with three digits, it appears that the addition of one digit to the square root, adds two digits to the square: hence in finding the square root by the method just to be described, we ascertain the number of figures the root will contain, by dividing the square into periods of two figures; a dot being placed on the unit, and on every third figure to the left hand, if the number be an integer.

CXXXVIII. **To extract the square root of any number.**

Having divided the number into periods of two figures, beginning at the unit's place; and if the number be partly integral and partly a decimal, going to the left in the integral part, and to the right in the decimal part:

1st. Find the greatest square in the left-hand period, place its root as the first figure of the root; place its square under the first period, and subtract; and add to the right of the difference, the next period.

2d. Double the root for a divisor, and find how often it

is contained in the dividend, omitting the unit: and place the result both in the root and in the divisor.

3d. Multiply the divisor thus augmented by the new figure of the root, and subtract the product from the dividend, and to the remainder add the next period.

4th. For a new divisor, double the first two figures of the root, and find the third figure of the root, as you have found the second.

If the number be a complete square, the remainder will be finally zero; but if not, add two cyphers to the number after the decimal point, and another figure, which will be a decimal, of the root may be found; and thus proceeding, the root may be found to any degree of accuracy.

Ex. (1.) Extract the square root of 81796.

```
 8̇1̇7̇9̇6̇ ( 286
 4
 ---
48 ) 417
     384
 ---
566 ) 3396
      3396
 ----
```

Ans. 286.

Here, having divided the number into periods, we see that the next less square to 8 is 4, and its root is 2; 4 is placed under 8, and 2 is the first figure of the root; the difference between 8 and 4 is 4; bring down the next period 17, and the dividend is 417; double 2 for a divisor, and write 4 for its first figure; divide 41 by 4, and by trial we see that 8 is the figure of the root; put 8 in the divisor and in the root; multiply 48 by 8, and subtract the product 384 from 417 [had we taken 9 for the quotient, the divisor would be 49 and 49×9, or 441, exceeds 417]; to the difference 33 annex the period 96; the divisor is now 28×2 or 56, and 33 divided by 5 gives a quotient 6, which on trial is found to be the number required; for adding 6 to 56 it becomes 566, and $566 \times 6 = 3396$.

Ex. (2.) Extract the square root of 317.21812.

$$
\begin{array}{l}
\dot{3}1\dot{7}.2\dot{1}8\dot{1}2\dot{0}\ (\ 17.810618 \\
1 \\
\hline
27\ )\ 217 \\
189 \\
\hline
348\ )\ 2821 \\
2784 \\
\hline
3561\ )\ 3781 \\
3561 \\
\hline
356206\ )\ 2202000 \\
2137236 \\
\hline
3562121\ )\ 6476400 \\
3562121 \\
\hline
35621228\ )\ 291427900 \\
284969824 \\
\hline
6458076
\end{array}
$$

EXAMPLES OF SQUARE ROOT.

The square root of—

(1.) 1521 is 39.
(2.) 2304 is 48.
(3.) 7744 is 88.
(4.) 9409 is 97.
(5.) 17689 is 133.
(6.) 56169 is 237.
(7.) 121801 is 349.
(8.) 298116 is 546.
(9.) 418609 is 647.
(10.) 877969 is 937.
(11.) 3755844 is 1938.
(12.) .3984016 is 1996.
(13.) 64.853 is 8.053.
(14.) 71.7409 is 8.47.
(15.) 35. is 5.9160798.
(16.) 6.49 is 2.5475478.
(17.) 18.21 is 4.267317.
(18.) .1947 is .441248.

The square root of

(19.) 1.3 is 1.140175. (20.) $\frac{3}{4}$ is .866025.

(21.) 14876.2357 is 121.96. (22.) 152399025 is 12345.

(23.) .0024 is .0489897. (24.) .00032754 is .01809.

(25.) 15241578750190521 is 123456789.

CXXXIX. When the root consists of integers and decimals some trouble may be saved by the following method.

Extract the root till you have found one more than half the required number of figures in the root, and for the rest divide the last remainder by its corresponding divisor.

Ex. (26.) Extract the square root of 17 to *six* places of decimals.

```
17.0000000000 ( 4.123
16.
---
81 ) 100
      81
     ---
822 ) 1900
      1644
     ----
8243 ) 25600
       24729
      -----
8246 ) 8710000 ( 1056
       8246
      -------
        46400
        41330
       ------
         50700
         49476
        ------
          1224
```

The root is 4.1231056, which is exact to the last decimal place of the root: the operation is carried one place further than the question demanded: and by adding cyphers the other terms may be found.

Had contracted divisors been made use of, the exact number of decimals required would be at once found: as follows:—

$$\begin{array}{r} 8246 \,)\, 8710000 \,(\, 105. \\ 825 \\ \hline 46 \\ 42 \\ \hline 4 \end{array}$$

and the root is 4.123105.

Ex. (27.) Extract the square root of 34876.4289 by the contracted way, to six decimal places.

Ex. (28.) Find the square root of 106.9298 to five decimal places.

Ex. (29.) Required the square root of $\frac{23}{7}$ to six decimal places.

THE CUBE ROOT.

CXL. The cube root of 1 is 1; the cube root of 1000 is 10; of 1,000,000, is 100; therefore the cube root of a number less than 1000 consists of one figure, of any number between 1000 and 1,000,000 of two places of figures, &c. If then a point be made over every third figure, the first point being on the units, the number of points will show the number of places in its cube root.

Rule for Extracting the Cube Root.

1. Point the given number, and if there be decimal places, make them periods of three each, by adding cyphers.
2. Find the cube root of the first period, or of the next less cube, and put it in the quotient as the first figure of the root.
3. Subtract the cube of this number, and to the remainder annex the next period, for a dividend.
4. Divide this dividend by three times the square of

the figure of the root; and the first figure of the quotient will be the second figure of the root.

5. Raise the number expressed by the two figures of the root to the cube, and subtract it from the first two periods on the left, and to the remainder annex the following or third period, for a new dividend, which divide by three times the square of the number expressed by the two figures of the root, as before, for a third figure of the root, and so proceed till the whole be finished.

Note.—The explanation of this Rule, as well as that for the extraction of the square root, can only be completely given by means of algebra. There is another rule for the cube root, which is more commonly known, but although it is less laborious in working than that just described, it is more intricate and burdensome to the memory.

Ex. Find the cube root of 76765625.

$$\begin{array}{rl} & \dot{7}\dot{6}\dot{7}\dot{6}\dot{5}\dot{6}\dot{2}\dot{5}\ (\ 425 \\ & 64 \\ \hline 3 \times 4^2 = 48\ ) & 12765\ (\ 2 \\ & 76765 \\ \hline (42)^3 = & 74088 \\ \hline 3 \times (42)^2 = 5292\ ) & 2677625\ (\ 5 \\ & 76765625 \\ \hline (425)^3 = & 76765625 \\ \hline \end{array}$$

EXAMPLES OF THE CUBE ROOT.

The cube root of

(1.) 405224 is 74.
(2.) 912673 is 97.
(3.) 3048625 is 145.
(4.) 41781923 is 347.
(5.) 86938307 is 443.
(6.) 849278123 is 947.

The cube root of

(7.)	2379270375	is 1335.
(8.)	769	is 9.1616..
(9.)	17	is 2.5712..
(10.)	41278.242816	is 34.56.
(11.)	.0001357	is .05138..
(12.)	$\frac{3}{8}$	is .72112...

MISCELLANEOUS EXAMPLES.

THE examples are purposely arranged without regard to their difficulty, experience having convinced the writer of them that such is the best method of ascertaining the real proficiency of the pupil.

The teacher may be advised to select ten or twelve questions in succession as a good exercise of the class.

EXAMPLES.

(1.) If 126 yards of cloth be exchanged for 3 hhds. of liquor at 6*s.* 8*d.* per gallon, what is the cloth per yard?

(2.) In 18 cwt. 3 qrs. 12 lbs. how many oz.? In 27 days 16 hours 5 minutes how many seconds? and in 32 miles 3 furlongs 160 yards how many inches?

(3.) Find the amount of 42 cwt. at 2*l.* 16*s.* 6*d.* per cwt.

(4.) Divide 118*l.* 13*s.* by 84; and 675*l.* 12*s.* 6*d.* by 138.

(5.) Also divide 1061 cwt. 2 qrs. by 28; and 375 miles, 2 fur. 7 poles 2 yards 1 ft. by 39.

(6.) Reduce $\frac{4}{5}$ of 6*d.* to the fraction of 5*d.*

(7.) Find the value of .41683428 of 2*l.*

(8.) Extract the square root of 41605800625, and the cube root of 17173512.

(9.) If 36 men working 8 hours a day for 16 days, dig a trench 72 yards long, 18 feet wide, and 12 feet deep, in how many days of 12 hours long, will 32 men dig a trench 64 yards long, 27 feet wide, and 6 yards deep?

(10.) Find the interest of 217*l.* 18*s.* for 3 years at $3\frac{1}{2}$ per cent.

(11.) What is the discount on 1050*l.*, due 6 months hence, discounting at 4 per cent.?

(12.) Find the compound interest of 760*l.* 10*s.* for 4 years, at 4 per cent.

(13.) If 116*l.* 3 per cent. stock, be given for 100*l.* $3\frac{1}{2}$ per cent. stock, find the gain or loss at the end of 7 years.

(14.) Bought 3 casks of raisins, each weighing 2 cwt. 2 qrs. 25 lbs., what will it come to at 5*l.* 1*s.* 8*d.* per cwt.?

(15.) Reduce 1*l.* 19*s.* $5\frac{3}{4}d.$ to the decimal of 5*l.*, and divide 144 by .0012.

(16.) If $\frac{5}{8}$ of a cwt. cost $4\frac{1}{9}l.$, what will $4\frac{1}{2}$ lbs. cost?

(17.) The length of a court-yard is 58 ft. 6 in., its breadth 54 ft. 9 in., what will the paving of it come to at $4\frac{3}{4}d.$ per yard?

(18.) Express in words, 6030; 604052; 900300805—in figures, seven millions, seven thousand, nine hundred, and seventy-seven; and one billion, ten millions, two hundred thousand.

(19.) Add together, 23298001; 9387209; 136784920; 38700907; 360063; 9758876—and eighteen millions four hundred and one; sixty-four thousand and sixty-four; eight hundred and ninety-nine.

(20.) Find the difference between 618537546, and 207812639; and from two millions, twenty thousand, nine hundred and thirty, take one million, nine thousand and six.

(21.) Multiply 95364739 by 132; and eighty millions, seven thousand, six hundred, by eight millions, seven hundred and sixty.

(22.) Divide 3784926474826 by 384365; and 37250360 by 1080.

(23.) Reduce 349*l.* 13*s.* $2\frac{3}{4}$*d.* to farthings.
Reduce 215*l.* 6*s.* 6*d.* to sixpences.
Reduce 3 lb. 10 oz. 7 dwt. 5 grs. to grains.
Reduce 94 miles 1 fur. 6 per. to perches.
Reduce 21168 farthings to guineas.
Reduce 1503697 pints to quarters.
Reduce 60 guineas to shillings, crowns, and pounds.

(24.) Add together, 493*l.* 2*s.* $8\frac{1}{2}$*d.* 347*l.* 14*s.* $3\frac{1}{4}$*d.* 729*l.* 19*s.* 5*d.* 672*l.* 5*s.* $8\frac{3}{4}$*d.* 548*l.* 10*s.* 3*d.* 217*l.* 12*s.* $8\frac{1}{2}$*d.* 974*l.* 16*s.* $7\frac{1}{4}$*d.* 146*l.* 5*s.* $0\frac{1}{2}$*d.*

(25.) From 23 cwt. 1 qr. 5 lb. take 17 cwt. 3 qrs. 22 lb.
From 3 tn. 2 hhd. 0 gl. 0 pt. take 1 tn. 3 hhd. 32 gl. 4 pt.

(26.) Multiply 43 a. 2 r. 33 p. by 49; and 3*l.* 15*s.* $3\frac{3}{4}$*d.* by 117.

(27.) Divide 2045*l.* 16*s.* $5\frac{1}{4}$*d.* by 4083; and 857 miles 96 yds. 2 ft. by 120.

(28.) If 4 yards of linen cost 14*s.*, what is the value of 26 yards?

(29.) If a farm of 375 acres be let for 401*l.* 11*s.* 3*d.* a year, what is the rent per acre?

(30.) What is the price for one year of the Home Friend, which is a penny per week? how many numbers can be got for 5*s.* 6*d.*? how many monthly parts, of four numbers each, can be got for 10*s.* 6*d.*?

(31.) Find the greatest common measure of 2772, 2520, 2394; and the least common multiplier of 54, 81, 63, 14.

(32.) Reduce $374\frac{54}{103}$ to an improper fraction.

Reduce $\frac{471283640}{284248}$ to a mixed quantity.

Reduce $\frac{9}{11}$, $\frac{7}{13}$, $\frac{12}{19}$, to a common denominator.

Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$, to the least common denominator.

Reduce $\frac{4}{5}$ of $\frac{7}{8}$ of $6\frac{1}{2}$ of $\frac{5}{26}$ of $\frac{1}{14}$ of 4 to a simple fraction.

(33.) Find the sum of $1\frac{1}{3}$, $\frac{8}{3}$ of $\frac{41}{34}$, $\frac{4}{5\frac{1}{10}}$.

Find the difference of $13\frac{3}{28}$ and $9\frac{11}{35}$.

Find the product of $\frac{3}{8}$, $2\frac{1}{16}$, $1\frac{1}{63}$, $3\frac{5}{87}$.

Find the quotient of 6347 by $2\frac{5}{4}$.

(34.) Reduce $\frac{3}{5}$ of $\frac{\frac{1}{4}-\frac{1}{9}}{\frac{1}{2}+\frac{1}{3}} + \frac{1}{18}$ of $\frac{\frac{2}{3}+\frac{3}{2}}{1+\frac{4}{9}}$ to its simplest form.

(35.) Reduce $\frac{4}{5}$ guinea to the fraction of a penny.

Reduce 18 minutes to the fraction of an hour.
Reduce 1 qr. 13 lb. $7\frac{1}{5}$ oz. to the fraction of a ton.

(36.) Find the value of $\frac{3}{7}l. + \frac{5}{6}$ of a guinea.

(37.) If $3\frac{5}{8}s.$ buy $1\frac{1}{12}$ yd. of cloth, how much may be bought for $6\frac{17}{18}l.$?

(38.) 31 tons 8 cwt. 13 lb. at 57*l.* 17*s.* 9*d.* per ton (Practice).

(39.) Express by vulgar fractions .8125, .00875, 13.816.

(40.) Find the sum of 346.1201, 24.00076, .004, 30.9.
Find the difference of 37.001 and 4.41967.
Find the product of 3706.205 and 34.005.
Find the quotient of 1 by .1, and .5 by .00725.

(41.) Express in a decimal form $\frac{4}{99}$, and reduce $.\dot{2}\dot{6}\dot{5}$ to a vulgar fraction.

(42.) Reduce 11*s.* 9$\frac{3}{4}$*d.* to decimal of a guinea; and find the value of .0625 barrel of beer.

(43.) Find the simple interest on 824*l.* 18*s.* 9*d.* for 10 months at 6 per cent.; and the amount of 550*l.* 10*s.* for 3$\frac{1}{2}$ years at compound interest at same rate.

(44.) Find the value of 106 at 3*l.* 15*s.* 2*d.* each, and 4,567 at 4*l.* 16*s.* 9$\frac{1}{2}$*d.* each.

(45.) Divide 68*l.* 11*s.* 10$\frac{1}{4}$*d.* by 161.

(46.) How often will a wheel 16 ft. 6 in. in circumference, turn in 202 miles?

(47.) If 1 cwt. 18 lb. cost 108*l.* 1*s.* 3*d.*, what will 4 cwt. 59 lb. cost?

(48.) Find the simple interest of 2,346*l.* for 9 years 7 months, at 4$\frac{3}{4}$ per cent.

(49.) Find the amount of 2,026*l.* 13*s.* 4*d.* for 146 days, at 3$\frac{3}{4}$ per cent.

(50.) What is the rate per cent. when the interest of 345*l.* is 93*l.* 3*s.* in 6 years?

(51.) If cloth be bought at 6*s.* 8*d.* per yard, at what price must it be sold to gain 15 per cent.?

(52.) If 87$\frac{1}{2}$ lb. of tea cost 19*l.* 1*s.* 3*d.*, at what must it be sold to gain 5*l.* by the whole, and what is the gain per cent.?

(53.) Find the compound interest of 2,666*l.* 13*s.* 4*d.* for 4 years, at 2$\frac{1}{2}$ per cent.

(54.) If a grocer mix 6 lb. at 1*s.* 3*d.*, 12 lb. at 1*s.* 6*d.*, 18 lb. at 1*s.* 9*d.*; what is the price of the mixture?

(55.) What is the discount of 63*l.* 15*s.* for 15 months, at 5 per cent.?

(56.) Find the square roots of 1522756 and 30712.5625.

(57.) A ladder 100 feet long, is placed against a building 100 feet high, so as to reach within 20 feet of the summit; how far from the base of the building was the foot of the ladder placed?

(58.) Extract the cube root of 41063625.

(59.) How many cubic feet of gas are there in a gaso-

meter 20 feet 10 inches long, 18 feet 10 inches high, and 12 feet 8 inches broad, and what is its value at $\frac{3}{8}$ *d.* per foot?

(60.) Reduce $\frac{3}{4}$ of 6*s.* 8*d.* to the fraction of 7*s.* 6*d.*, and $\frac{7}{16}$ of a moidore to the fraction of a guinea.

(61.) Find the sum of $\frac{3}{4}$ of a shilling, $\frac{7}{8}$ of a crown, and $\frac{5}{14}$ of a guinea.

(62.) Reduce 4*s.* 6$\frac{3}{4}$*d.* to the decimal of a crown, and find the value of .425 of a guinea.

(63.) If 250 men dig a trench 450 yards long, 15 feet wide, and 5 feet deep in 21 days, how many will dig a trench 360 yards long, 12 feet wide, and 3 feet deep in 14 days?

(64.) Find in lowest terms the 5th power of $\frac{14}{21}$.

(65.) Find the 3d power of 1.04.

(66.) Find the number of solid feet in a cubic block of ice whose edge measures 12 feet.

(67.) Extract the square root of 8118190201.

(68.) Find the side of a square, equal to a rectangle whose dimensions are 16 ft. and 144 ft.

(69.) Required the cube root of 13 to three places of decimals.

(70.) The area of a circle being 3.1416 times the square on the radius, find, to two places of decimals, the length of the tether when a donkey has an acre to feed upon. N.B. 4840 square yards = 1 acre.

(71.) Required the simple interest on 193*l.* 11*s.* 3*d.* for 3 years, 11 months, at 4$\frac{1}{8}$ per cent.

(72.) Find the net value of 1 ton, 8 cwt. 2 qrs. 8 lbs. at 5*s.* per lb., after deducting 2$\frac{1}{2}$ per cent. from the weight, and then 5 per cent. from the value of the diminished weight.

(73.) Find the *amount* of 537*l.* 16*s.* 8*d.* in 4 years, at $2\frac{1}{2}$ per cent. simple interest.

(74.) Required the simple interest on 193*l.* 11*s.* 3*d.* for 3 years 11 months, at $4\frac{3}{8}$ per cent.

(75.) Required the discount on 177*l.* 12*s.* 6*d.* payable at the end of 4 months, at $4\frac{1}{2}$ per cent.

(76.) Find the commission on 384*l.* 17*s.* 9*d.* at 2 per cent.

(77.) Find the brokerage on 731*l.* 17*s.* $9\frac{1}{2}d.$ at $\frac{1}{4}$ per cent.

(78.) Find by Practice the values of—

59,623 at $7\frac{3}{4}d.$ each,
38,940 at $11\frac{3}{4}d.$ each, and
1,356 at 7*l.* 15*s.* $7\frac{3}{4}d.$ each.

(79.) Also, by the same method, find the value of—

237 cwt. 2 qr. 14 lbs. at 5*l.* 10*s.* 9*d.* per cwt., and
3 cwt. 2 qr. 7 lbs. at 3*s.* 10*d.* per lb.

(80.) A gentleman hires lodgings at $2\frac{1}{2}$ guineas per week; what must he pay if he occupy them for 3 months and 15 days?

(81.) What is the amount of income-tax on an annuity of 500 guineas, the tax being 7*d.* in 1*l.*?

(82.) If the carriage of 3 cwt. 1 qr. 18 lbs. of goods be 18*s.* $9\frac{1}{2}d.$; what will be the cost of the carriage of a ton?

(83.) Find the sum of—

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}, \text{ and of } \frac{1}{3} + \frac{1}{5} + \frac{1}{7}.$$

(84.) Find the value of $\frac{367}{1296}$ of a moidore, and of

$$\frac{2}{5} \text{ of } \frac{3}{7} \text{ of } 16l.\ 8s.\ 1\tfrac{1}{2}d.$$

(85.) Divide 5 cwt. 2 qrs. 14 lbs. by $3\frac{1}{3}$.

(86.) Reduce 15*s.* $9\frac{1}{2}d.$ to the fraction of 1*l.*, and 13 weeks, 5 days, $6\frac{1}{2}$ hours, to the fraction of $365\frac{1}{4}$ days.

(87.) Compare $\frac{1}{19}$ of 1*l.*, $\frac{1}{20}$ of a guinea, and $\frac{8}{35}$ of a crown.

(88.) Reduce $\frac{7}{13}$ of half-a-crown to the fraction of half-a-guinea, and 10*s.* $1\frac{1}{2}$*d.* to the decimal of 1*l.* 5*s.*

(89.) Divide $2\frac{1}{8}$ by $\frac{3}{5}$ of $4\frac{1}{3}$, and 14.4 by .0012 and by 120.

(90.) Prove that $\frac{3\sqrt{8}-2\sqrt{7}}{\sqrt{8}-\sqrt{7}} = 2.51...$ and find $\sqrt[3]{1010}$ to four places of decimals.

(91.) Add together $\frac{3}{4}$ of 6*s.* 8*d.* and $\frac{2}{7}$ of a guinea, and reduce the sum to the decimal of 100*l.*

(92.) Find the amount of 400*l.* in 3 years and 35 days, at $3\frac{5}{4}$ per cent. per annum.

(93.) Find the greatest common measure of 837, 1134, and 1377.

(94.) Extract the square roots of 826281 and 1.02045.

(95.) Divide 35 by .0175, and .0175 by 35.

(96.) Divide 199*l.* 3*s.* 10*d.* by 53.

(97.) What is the cost of 60 cwt. 3 qrs. 12 lbs. at 7*l.* 13*s.* 6*d.* per cwt.?

(98.) Add together $100\frac{2}{5}$, $64\frac{5}{9}$, and $\frac{3}{5}$ of 701.

(99.) Multiply 2.43 by .012, and divide 3.46 by 9.051 to four decimal places.

(100.) Find the value of .215 of a guinea, reduce 4*s.* $9\frac{3}{4}$*d.* to the decimal of 1*l.*

(101.) Find the amount of 690*l.* for 3 years at $4\frac{1}{2}$ per cent. per annum.

(102.) Reduce $\frac{825}{960}$ to its lowest terms, and extract the square roots of 11881 and 51.84.

(103.) A person's daily income is 1*l.* 5*s.*, and his quarterly expenditure is 135*l.* 10*s.*; how much will he be in debt at the end of two years and a quarter?

(104.) Find the greatest common measure of 805, 1311, and 1978.

(105.) Extract the square roots of 824464 and 1.03054.

(106.) What is the discount of 340*l.* due 5 months hence, at 4 per cent. per annum?

(107.) Find the value of 23 yds. 1 ft. 7 in. at 4*s.* 5*d.* per foot.

(108.) Find the amount of 136*l.* for 4 years, at $4\frac{1}{2}$ per cent. per annum.

(109.) From $7\frac{2}{3}$ take $5\frac{3}{4}$; reduce $\frac{3}{13}$ of $2\frac{1}{2}$ to a simple fraction, and $\frac{2484}{2628}$ to its lowest terms.

(110.) Multiply 1.23 by .0059, and divide 600 by .005.

(111.) Find the square root of .5 to 3 places of decimals, and divide twice the square root of 3 by the cube root of 2.

(112.) If the discount of 150*l.* for $1\frac{1}{2}$ years be 12*l.*; find the rate of interest.

(113.) 100*l.* is payable at the end of 6 months, and 150*l.* at the end of 9 months; find the equated time of payment.

(114.) Reduce 18*s.* $4\frac{1}{2}$*d.* to the decimal of 1*l.*, and 3 qrs. 3 lbs. 1 oz. 12 drs. $\frac{4}{9}$ to the decimal of a cwt.

(115.) Find value of .03 of a shilling, .87812 of 1*l.*, and .375 of a guinea.

(116.) Find the product of 15 ft. 7 in. by 5 ft. 11 in.

(117.) What is the interest of 1,643*l.* 7*s.* $5\frac{1}{2}$*d.* for 4 years, at $3\frac{1}{2}$ per cent.?

(118.) Extract the square roots of 43046721 and of 876.535, and the cube root of 134217728.

(119.) If 24 men in 6 days of 10 hours each, can dig

1,600 yards of earth, in how many days of 12 hours each can 72 men dig 2,560 yards?

(120.) What is the price of a marble slab, whose length is 10 feet 11 inches, and breadth 3 feet 9 inches, at 5*s.* 6*d.* per foot?

(121.) What must be paid for 1,265*l.* 8*s.* 3*d.* 3 per cent. stock, at 97$\frac{3}{4}$ per cent.

(122.) What portion of a florin is 1*s.* 7$\frac{1}{2}$*d.*; find value of .027865*l.*

(123.) Reduce 5,427*l.* to guineas,
292571 grs. troy to lbs., and find in
491256 seconds how many days.

(124.) Multiply 2*l.* 19*s.* 5$\frac{1}{4}$*d.* by 1845, and
27 cwt. 3 qr. 12 lb. by 39.

(125.) Divide 1,394*l.* 7*s.* 10*d.* by 97, and
170*l.* 16*s.* 8*d.* by 2*l.* 1*s.* 8*d.*

(126.) If 31 lbs. of tea cost 7*l.* 15*s.* what will 6 cwt. 1 qr. 6 lbs. cost?

(127.) The poor rate of a parish is 196*l.* 17*s.* 6*d.*; the rental is 3,500*l.*; what is the assessment per 1*l.*?

(128.) A has $\frac{2}{3}$ of a mine, and sells $\frac{5}{12}$ of his share for 300*l.*; what is the value of the mine?

(129.) By practice, find the value of
43,253 at 16*s.* 8*d.* each,
and 1,234$\frac{5}{6}$ at 1*l.* 16*s.* 8$\frac{1}{2}$*d.* each.

(130.) Find the value of .67,352,718*l.* and 1.92,834,906*l.*

(131.) Find the interest of 1,739*l.* from May 5th to Nov. 20th, at 2$\frac{1}{2}$ per cent. per annum.

(132.) Find the square root of 62,504,836, and the cube root of 843,908,625.

(133.) Find the product of 1.3038404 by 1.3038404 and the square root of 1.7 to 7 places of decimals.

(134.) How much silk $\frac{7}{8}$ yard wide will line a cloak 1 yd. 3 qrs. long, and 9 qrs. 3 nails broad?

(135.) If the freight of a ship of 500 tons for 3 months be 245*l.*; what would be the freight of a ship of 230 tons for 7 months?

(136.) If 120 men can dig a trench 150 yds. long, 2 yds. deep, and 3 yds. broad, in 8 days of 12 hours each, in how many days will 50 men dig a trench 80 yds. long, 1 yd. deep, and 2 broad?

(137.) Multiply 61.032572 by 1.6769235 so as to retain 4 places of decimals in the product.

(138.) Divide 26.5234 by 42.6792, and have 6 figures in the quotient.

(139.) A window is 7 ft. 8 in. high, and 3 ft. $6\frac{1}{2}$ in. wide; what will be the expense of glazing it at 1*s.* 7*d.* a square foot?

(140.) Find the interest and the amount of 125 guineas for $3\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. compound interest.

(141.) Extract the square roots of 1.21 and 12.1, and the cube root of 84604519.

(142.) If when wheat is 16*s.* 3*d.* a bushel the penny loaf weigh $\frac{2}{3}$ of $\frac{5}{6}$ of 18 oz.; find what it ought to weigh, when a bushel of wheat is worth $\frac{2}{7}$ of 13*s.* 6*d.*

(143.) Two merchants, *A* and *B*, insure a ship's cargo; *A* has ventured 1,520*l.*, for which he pays $\frac{1}{2}$ of $\frac{6}{7}$ of the insurance, which amounts to 420*l.*—Find what *B* paid of the insurance, and what he ventured in the ship.

(144.) What number of fifty pound shares, bearing a premium of $12\frac{1}{2}$ per cent. must be given for 30 fifty pound shares, which are at a discount of $17\frac{1}{2}$ per cent.?

(145.) Find how many yards of carpet, $\frac{3}{4}$ of a yard wide, will be sufficient to cover a room, the length of which is 30 ft., and the breadth 18 ft.

(146.) A man sells a horse for 24*l.* 8*s.*, and loses

26*l.* 16*s.* per cent. on what the horse cost him; find what he gave for the horse.

(147.) If the carriage of 30 cwt. of baggage for 20 miles cost 1*l.* 13*s.*; what must be paid for the carriage of 45 cwt. for 35 miles?

(148.) In how many days of 10 hours each, will 25 men reap a field, containing 36 acres, when 15 men, in 5 days, of 8 hours each, can reap a field containing 20 acres?

(149.) What quantity of water must be added to a pipe of wine, which cost 75*l.* to gain 25 per cent. by selling it at 12*s.* a gallon?

(150.) Find the cost of carving a cubical rosewood chest, one of whose edges is 2 ft. 6 in. at the rate of 7*s.* 6*d.* a square foot.

(151.) If the French foot be $6\frac{1}{2}$ per cent. greater than the English foot, and if the metre be 39,37 English inches, how many metres are there in 1,000 French feet?

(152.) Add together $\frac{4}{5}$, $\frac{6}{7}$, and $\frac{9}{10}$.

(153.) Prove that the sum of $5\frac{1}{3}$ and $3\frac{1}{5}$ is equal to four times their difference.

(154.) Multiply $2\frac{3}{8}$ by $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{7}{9}$.

(155.) Divide $3\frac{4}{9}$ by $\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{5}{6}$.

(156.) Reduce $\frac{3}{32}$, cwt. to the fraction of a lb.; and $\frac{4}{7}$ of an ounce to the fraction of a cwt.

(157.) Change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$ into equivalent fractions having the least common denominator.

(158.) What are the decimal fractions equivalent to $\frac{4}{25}$, and $\frac{9}{125}$?

(159.) Find the sum of .295, 3.086, 12.87, .0051, and 729.54.

(160.) What is the product of 275, 2.75, and 27.5?

(161.) Express 12*s.* $6\frac{3}{4}d.$, and 15*s.* $9\frac{3}{4}d.$, as decimals of 1*l.*

(162.) Find the value of 2.36875*l.*, and of 13.3375 acres.

(163.) Extract the square root of 998001, and the cube root of 117649.

(164.) Find the price of the carpet of a room 18 ft. 6 in. long, and 14 ft. 3 in. broad, at 5*s.* per square yard.

(165.) If 10 men reap 20 acres of corn in 4 days, how many men can reap 70 acres in 10 days?

(166.) A staff $5\frac{5}{6}$ ft. high casts a shadow $6\frac{5}{12}$ ft., what is the height of a steeple which casts a shadow $220\frac{1}{2}$ ft.?

(167.) Calculate the interest on 250*l.* for 9 months, at 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, and 6 per cent. per annum.

(168.) A ship worth 900*l.* being entirely lost, of which $\frac{1}{8}$ belonged to *A*, $\frac{1}{4}$ to *B*, and the rest to *C*, what loss will each sustain, supposing 540*l.* of her value to have been insured?

(169.) If $\frac{7}{11}$ of a house cost 360*l.*, what will $\frac{17}{24}$ of the same cost?

(170.) How many planks, 15 ft. long, and $1\frac{1}{4}$ wide, will floor a room $60\frac{1}{2}$ ft. long, and $33\frac{1}{2}$ wide?

(171.) What will 500*l.* amount to in 3 years, at 5 per cent. per annum, compound interest?

(172.) Reduce $\frac{4}{5}$, $2\frac{5}{8}$, and $3\frac{9}{11}$ to fractions having a common denominator.

(173.) What is the value of $3\frac{5}{8} + 2\frac{3}{5} + \frac{7}{11} + 7\frac{1}{6}$?

(174.) Find the difference of $1\frac{2}{3}$ of $3\frac{5}{8}$, and $2\frac{7}{9}$ of $16\frac{2}{5}$.

(175.) What is the product of $2\frac{3}{8}$ by $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{7}{9}$.

(176.) Divide $15\frac{7}{11}$ of $8\frac{4}{3}$ by $\frac{4}{5}$ of $\frac{6}{11}$ of $15\frac{5}{6}$.

(177.) What is the value of

$$\frac{2247}{1017} \div \frac{903}{1107} \times \frac{774}{615} \div \frac{1926}{565}.$$

(178.) If $\frac{3}{16}$ of a lottery ticket cost 4*l.* 10*s.*, what is the price of $\frac{1}{5}$ of $\frac{1}{4}$ of a ticket?

(179.) A person possessed of $\frac{2}{5}$ of a coal mine, sells $\frac{3}{4}$ of his share for 2,000*l.*; what is the value of the whole mine?

(180.) If a piece of work can be performed in $17\frac{1}{2}$ days, by working $13\frac{3}{4}$ hours per day, how many days will be required if $10\frac{1}{8}$ hours per day be employed?

(181.) If 75*l.* be due in 4 months, 125*l.* in 5 months, and 150*l.* in 7 months, what is the equated time for the payment of the whole?

(182.) What will 387*l.* 17*s.* 9*d.* amount to in $2\frac{3}{4}$ years at compound interest, at $4\frac{1}{2}$ per cent. per annum?

(183.) I sell cloth at 8*s.* 9*d.* per yd. and gain 12 per cent.; what would be the gain per cent. if I sold it at 10*s.* 6*d.* per yd.?

(184.) A father left his estate, valued at 19,090*l.*, among his three sons, so that for every 2*l.* *A* receives, *B* shall have 3*l.*, and *C* 5*l.* How is the estate to be divided?

(185.) Extract the square root of 22.9441.

(186.) Extract the cube root of 46656.

(187.) What is the price of a marble slab the length of which is 5 ft. 7 in., and the breadth 1 ft. 10 inches, at 6*s.* per ft.?

(188.) How many francs can I receive for 140*l.*, exchange at 25.75 francs per 1*l.* sterling?

(189.) Change 152,246 oz. to tons, 73938 in. to miles, 1,234*l.* 17*s.* 6*d.* into sixpences, and 16 yds. 20 ft. 1,456 in. (solid measure) into inches.

(190.) Required the price of 659 cwt. at 4*l.* 11*s.* 5*d.* per cwt.

(191.) If 47 yds. cost 34*l.* 10*s.* $3\frac{3}{4}$*d.*, what will 11,738 cost?

(192.) If 18 men can build a wall in 28 days, how many will build it in 7 days?

(193.) If 19 galls. cost 8*l.* 1*s.* 6*d.*, how many galls. may be purchased for 243*l.* 2*s.*?

(194.) If the carriage of 14 cwt. 0 qr. 23 lbs. for 65 miles cost a certain sum, what weight will be carried for the same sum 37 miles?

(195.) Required the neat weight of 51 barrels, each weighing 1 cwt. 0 qr. 13 lbs., allowing $4\frac{2}{3}$ lbs. per bag for tare.

(196.) Required the neat weight of 14 casks, each weighing 1 cwt. 0 qr. 23 lbs. 8 oz., tare at $12\frac{1}{2}$ lbs. per cwt. and tret 1 lb. per cask.

(197.) Required the amount of 325*l.* 16*s.* 8*d.*, at $4\frac{1}{2}$ per cent. simple interest, in $3\frac{1}{2}$ years.

(198.) Required the amount of 550*l.* 10*s.* at 4 per cent. compound interest, in $1\frac{1}{2}$ year, interest being paid *half-yearly*.

(199.) Required the commission on 975*l.* 10*s.* 3*d.* at 5*s.* 6*d.* per cent.

(200.) Required the interest on 641*l.* 12*s.* 6*d.* for 36 days at $4\frac{1}{2}$ per cent.

(201.) Required the present worth of 241*l.* 12*s.* 4*d.* due at the end of 146 days, at $4\frac{1}{3}$ per cent.

(202.) Required the neat weight of 14 hhds., each weighing 7 cwt. 2 qrs., after allowing tare at 7 lbs. per cwt. and tret $\frac{1}{26}$ of remainder.

(203.) Required simple interest on 273*l.* 19*s.* 10*d.* for $3\frac{1}{2}$ years at 3 per cent.

(204.) Required simple interest on 1,000*l.* for 1 year $3\frac{1}{2}$ months at 5 per cent.

(205.) Required amount of commission on 3,501*l.* 1*s.* at $\frac{11}{16}$ per cent.

(206.) Required present worth of 1,642*l.*, due 9 months hence, discounting at $3\frac{1}{2}$ per cent.

(207.) How much tobacco at 5*l.* 5*s.* per cwt. must be given for 6 cwt. 1 qr. 14 lbs. of snuff at 4*s.* 6*d.* per lb.

(208.) 1*l.* 12*s.* is paid per cwt.; on this 25 per cent. is to be gained; required price per cwt., and the quantity to be sold to gain 100*l.*

(209.) A horse costs 22*l.*; at what must it be sold to gain 30 per cent. upon this sum?

(210.) Required the weight of a certain priced loaf when wheat is at 56*s.* per qr., if its weight is 3 lbs. 2 oz. when wheat is at 50*s.*

(211.) Find the value to a fraction of one penny of

$$\frac{5}{12}l. + \frac{2}{7}s. + \frac{5}{6}\text{guin.}$$

(212.) If a man can complete a piece of work in 4 days, and a boy the same in 9 days; what part of the work will they accomplish in 2 days, working together?

(213.) Find the continued product of—

$$.00259 \times 3.817 \times .00615 \times 21.39.$$

(214.) Required the surface and vol. of a rectangular block 7 ft. 4 in. long, 3 ft. 3 in. broad, 2 ft. 6 in. deep.

(215.) Extract the square root of 5 to 6 places of decimals.

(216.) Find the number whose square $= 2 \times 23^2 + 23 + 8$.

(217.) 170*l.* amounted to 182*l.* 15*s.* at 5 per cent. simple interest; find the time.

(218.) Find the value in francs of 675*l.* 18*s.* 3*d.* at 23 fr. 15 cents. per 1*l.*

(219.) By selling goods at 8*s.* I lost 16 per cent.; at what rate must I sell them to gain 7 per cent.?

(220.) A merchant had $5\frac{3}{8}$ cwt. of sugar at $6\frac{1}{4}$ per lb., which he bartered for tea at $8\frac{1}{8}s.$ per lb. How much tea did he receive for the sugar?

(221.) Two men travelled from the same town, the one north 28 miles per day, the other west 36 miles per day. How far were they distant after travelling 6 days?

(222.) A puts in 180$l.$
B — 240$l.$
C — 350$l.$
D — 430$l.$
} They gain 3,600$l.$ Required each man's share of this gain.

(223.) Three sorts of coffee are mixed: viz. 50lbs. at 12$d.$
44lbs. — 9$d.$
28lbs. — 8$d.$
Required the worth of the mixture per lb.

(224.) If 100$l.$ is due in 9 months,
500$l.$ — 15 —
} Find the equated time.

(225.) A has 7,800 lbs. of pepper at 1$s.$ $3\frac{1}{2}d.$ per lb., B has ginger at $7\frac{1}{2}d.$ and cinnamon at 4$s.$ 9$d.$ per lb. A exchanges the pepper for equal quantities of ginger and cinnamon. How many lbs. of each does he receive?

(226.) If needles be bought at 2$s.$ 6$d.$ a gross, how many may be sold for 1$d.$ to gain 20 per cent. upon the cost price?

(227.) A, B, C, continue in trade for a year with a joint stock of 700$l.$ At the end of that time A's share of the gain was 25, B's 48, C's 27; required each partner's stock.

(228.) $\frac{1}{4}$ of a debt is due at Lady-day,
$\frac{1}{4}$ — Midsummer,
$\frac{1}{4}$ — Michaelmas,
$\frac{1}{4}$ — Christmas.
How many days after Lady-day should the whole be paid?

(229.) Reduce 4,560 francs 50 centimes into sterling: exchange at 23 francs 75 centimes per £ sterling.

(230.) Reduce 580$l.$ sterling into marks banco: exchange at 13 marks 6 schillings per £ sterling. Note, 16 schillings = 1 mark.

(231.) Explain the difference between interest and dis-

count, and calculate the discount on 500*l.* for 2 months, and 375*l.* for 90 days, at 5 per cent., and at 6 per cent.

(232.) State what is meant by insurance, commission, and brokerage: and calculate the insurance on 1,500*l.*, at $3\frac{1}{2}$ per cent.; brokerage on 850*l.*, at $\frac{1}{2}$ per cent.; and commission on 1,250*l.*, at $2\frac{1}{2}$ per cent.

(233.) Reduce—

$\frac{18}{32}$ and $\frac{5}{7}$ to decimal fractions; and

.009101, and .1001 to vulgar fractions.

(234.) Perform the following operations:—

19.876 — 3.042361, and 101.0 — .0019.

1.1 × .011 × 2.02 × .002, and 4·234 ÷ .00365.

(235.) Find the value of 22.75 of 5*l.* 10*s.* 6*d.*; and reduce 3 hrs. 3′ $2\frac{1}{4}''$ to the dec. of 1 day.

(236.) If the circumference of a circle = 3.1416 times the diameter, find the radius of the earth, whose circumference is 24857 miles.

(237.) Required the area of a pavement whose length is 23 ft. 11 in., and breadth 3 ft. 4 in.

(238.) Required the content of a chamber 71 ft. 2 in. long, 34 ft. 5 in. broad, 20 ft. 3 in. high.

(239.) Can $\frac{12}{15}$ be exactly represented by a decimal fraction?

(240.) Express .0009375 by a vulgar fraction in its lowest terms.

(241.) Find the values of—

1.0000123 + 31.1 + 117.154 + 2343.008 + ·0002.

3.45 — .00098.

.003 × .01 × 500000.

1.1 × .011 × 1.01 × .0101.

4 ÷ .00255.

$.2\dot{3} \times .\dot{3}\dot{6}$.

(242.) If 1 lb. of sugar cost .0703125 of 16*s.*, what is the cost of .0625 cwt.?

(243.) Find, by duodecimals, the expense of paving a yard which is 23 ft. 6 in. by 13 ft. 9 in., at 4*s.* 6*d.* the square yard.

(244.) Find the simple interest and amount of 355*l.* 15*s.* for 4 years, at 4 per cent. per annum.

(245.) What must be paid to a stock-broker for buying 360*l.* stock in the three-per-cents. reduced, the commission being $\frac{1}{8}$*l.* per cent.

(246.) A cargo valued at 3,561*l.* 11*s.* 7$\frac{1}{2}$*d.*, being insured at 8$\frac{1}{3}$ per cent.—what is the insurance?

(247.) If I pay an agent 3$\frac{1}{4}$ per cent. for selling goods to the amount of 240*l.* 16*s.* 8*d.*, what is his commission?

(248.) What is the discount of 45*l.* 12*s.* 6*d.* for 56 days at 5 per cent. per annum?

(249.) What is the net weight of 32 cwt. 3 qrs. 12 lbs. of sugar, tare 14 lbs. per cent., and tret 4 lbs in 104 lbs.?

(250.) What is the net weight of 6 hogsheads of tobacco, each 9 cwt. 1 qr. 14 lbs., tare 3 qrs. 18 lbs. per hogshead, tret 4 lbs. in 104, and cloff 2 lbs. in 336?

(251.) 170*l.* amounted to 182*l.* 15*s.* at 5 per cent. simple interest—find the time.

(252.) Required the value of a cube whose edge = .05 feet.

(253.) Find the 5th power of .01, and the 4th power of $\frac{14}{35}$.

(254.) Extract the square roots of 717409, of 5 to 6 places of decimals, and of $\frac{2401}{14641}$.

(255.) Extract the cube roots of $\frac{373248}{110592}$, and of 24 to 3 places of decimals.

(256.) Find the number, whose square
$$= 2 \times 23^2 + 23 + 8.$$

(257.) If the base of a right-angled triangle is 342 feet, and the perpendicular 475 feet: required the length of the hypothenuse.

(258.) Reduce to a single term:—

1. $31.01 + .037 + .00615 + 27.382$;
2. $101.02 - 1.00900$;
3. $.00259 \times 3.817 \times .00615 \times 21.39$;
4. $51.2 \div 613.912$.

(259.) Reduce $\frac{7}{1001}$ to a decimal fraction; and 1 rood, 10 poles, to the decimal of 1 acre.

(260.) Determine (to seconds) the value of .242256 day.

(261.) How many square feet of flooring in a room 24ft. 7in. by 16ft. 4in.

(262.) Required the surface and volume of a rectangular block, which is 7ft. 4in. long, 3ft. 3in. broad, and 2ft. 6in. deep.

(263.) Define those vulgar fractions which can be exactly expressed in the decimal form. Convert $\frac{177}{625}$ and $\frac{5}{16}$ into decimals *without* division.

(264.) From 3.45 take .00098.

(265.) Find the continued product of

$$.102 \times 1.02 \times 10.2;$$

and prove the truth of your result by using vulgar fractions.

(266.) Divide 181.3 by .00037.

(267.) Reduce 3 furlongs 33 yards to the decimal of 1 mile; 12h. 55′ 21″ to the decimal of 1 day; and find the value of .176 of 1 furlong 36 poles 2 yards 5 inches.

(268.) What length of that which is 2ft. 3in. wide will be required to cover a square whose side = 18ft. 9in.; and what will be the cost, at 2*s.* the linear yard?

(269.) Reduce to single fractions, $\frac{1}{4} + \frac{3}{8} + \frac{5}{6} + \frac{23}{24}$;

$12\frac{1}{2} + 24\frac{3}{4} + 17\frac{5}{8} - 54\frac{7}{8}$; $12\frac{1}{2} \times 23\frac{2}{3}$;

and $1234567 \div 26\frac{2}{15}$.

(270.) Find the value to a fraction of one penny; of $13\frac{4}{11}l. - 8\frac{7}{12}l.$

(271.) Find the value of 845 yards at $7l.\ 11s.\ 3\frac{3}{4}d.$, and of 745ft. $10\frac{1}{2}$in. at $4s.\ 9\frac{1}{2}d.$ per foot.

(272.) Multiply the 3d power of $\frac{9}{4}$ by the 6th power of $\frac{2}{3}$.

(273.) Required the square roots of 390141504: of 1.7 to 8 places of decimals: and of $\frac{3}{8}$ to 9 places of decimals.

(274.) The paving of a circular space 50 feet in diameter costs $74l.\ 16s.\ 8d.$; what will the paving of another cost, which is 120 feet in diameter?

(275.) Extract the cube roots of 843908625, and of 24 to 4 places of decimals.

(276.) *A*'s stock of 340*l.* was 4 months in trade,
B's stock of 510*l.* was 8 months in trade,
C's stock of 850*l.* was 10 months in trade,
the gain amounts to $270l.\ 13s.\ 6d.$; required, each one's share of the gain.

(277.) *A* and *B* entered into partnership for two years. *A* contributing 960*l.*, and *B*, 1,500*l.*; after 8 months they admit *C*, with a capital of 720*l.*; at the end of the period the gain amounts to $847l.\ 15s.$; required, the share of each.

(278.) If 100*l.* are due at the end of 2 months, and 750*l.* at the end of 7 months; what is the just time for the payment of both debts?

(279.) 28lbs. of tea, which cost $4s.\ 9d.$ per lb. are mixed with 56lbs. which cost $5s.$ per lb.; required, the price per lb. at which the mixture should be sold to gain $3d.$ on every lb.

(280.) Required the value of 65 Napoleons, if 1 Napoleon = 20.8fr., and $1l. = 25.45$fr.

(281.) Required the value of 72 thalers; if 1 thaler = 30 groschen, and $1s. = 10.1$ groschen.

(282.) Raise $\frac{5}{15}$ to the 4th power, .0012 to the 2d power, and 1.01 to the 3d power.

(283.) Find the square root of .01595169, the cube root 16934.994432, and the square root of $7\frac{9}{16}$.

(284.) Reduce 234a. 3r. 27p. to poles; 2,174 ounces of gold, to pounds; 365 days, 5 hours, 48 minutes, 51 seconds, to seconds; and 782,512 solid inches to solid yards.

(285.) Required the cost of 3cwt. 0qr. 31lbs., if 31lbs. cost 7*l.* 15*s.*; and the cost of 33 hogsheads, if 41 cost 287*l.* 2*s.* $6\frac{3}{4}d.$

(286.) Find the distance a man will walk in 4 hours, if he walk 7 miles in 2 hours 10 minutes.

(287.) In what time will 64 men build a house, if 57 can build it in 108 days?

(288.) Find the weight of a certain priced loaf, when wheat is at 56*s.* per qr.; if its weight is 3lb. 2oz. when wheat is at 50*s.*

(289.) If 1*s.* worth of bread weigh 6lbs. when wheat is at 6*s.* a bushel; what ought it to weigh when wheat is at 6*s.* 9*d.* a bushel?

(290.) Reduce these fractions to their *least* Common Denominator:—

$$\frac{3}{5}, \frac{7}{10}, \frac{6}{25}, \frac{11}{30}, \frac{13}{45}, \frac{23}{60}.$$

(291.) Find the value of 7cwt. 3qrs. 11lbs. at 2*l.* 13*s.* 1*d.* per qr.

(292.) Find the continued product of $.102 \times 1.02 \times 10.2$; and prove the truth of your result by using vulgar fractions.

(293.) Required the cubic feet of timber in a beam, whose dimensions are 20 feet long, 3 feet broad, and 2 feet 6 inches deep.

(294.) Find the distance between two people, who have travelled from the same point, in directions due north and due east, 80 and 60 miles respectively.

(295.) Find the *amount* of 500*l.* from March 1 to January 9, at $4\frac{5}{8}$ per cent.

(296.) Find the *amount* of 50*l.* for three years, at 5 per cent. compound interest.

(297.) Find the true discount upon a bill of 419*l.* 12*s.* 1*d.*, drawn March 6th, at 7 months, and discounted September 15th, at 5 per cent.

(298.) 11,000*l.* are transferred from the 4 per Cents. at 92 to the 5 per Cents. at 110, (both prices including the brokerage,)—what is the difference of income?

(299.) Two horses, whose velocities are respectively
$186\frac{1}{4}$ miles in $16\frac{2}{3}$ hours,
and $196\frac{7}{8}$ miles in $18\frac{2}{3}$ hours,
start at the same instant from the same point, in opposite directions; compare their rates, and find how far they will be separated in $6\frac{3}{4}$ minutes.

(300.) Find the greatest Common Measure and Least Common Multiple of 385 and 326.

(301.) Reduce these fractions to their *least* Common Denominator:

$$\frac{3}{5}, \frac{7}{10}, \frac{6}{25}, \frac{11}{30}, \frac{13}{45}, \frac{23}{60}.$$

(302.) Find the value of—

$$5\frac{3}{4} + \frac{3}{5} \text{ of } 7\frac{1}{2} + 8\frac{3}{10}; \text{ and of}$$

$$5\frac{1}{3} \text{ of } 4\frac{1}{2} - 3\frac{1}{4} \text{ of } 3\frac{1}{5}.$$

(303.) Reduce $3\frac{1}{7}$ of 1 cwt. 3 qrs. to the fraction of 1 ton.

(304.) If one yard of lace cost $1\frac{2}{3}\frac{9}{6}$*l.* what will $16\frac{1}{2}\frac{1}{5}$ yards cost?

(305.) Find the value of—

37 at 5*l.* 17*s.* 6*d.* each;

and of 111 at 18*s.* $7\frac{1}{2}$*d.* each.

(306.) Multiply 439 by 237, and give a reason for the position of each figure.

(307.) How much must be divided among 54 men, in order that each may receive 7*l.* 3*s.* 4¼*d.*

(308.) If 27 oxen can be purchased for 593*l.* 1*s.* 5½*d.*, what is the average price of each?

(309.) How many coins, each worth 9*s.* 6*d.*, are contained in 231*l.* 16*s.*?

(310.) Change—

3700 half-crowns into fourpenny pieces;
36*l.* 17*s.* 6*d.* into crowns; and
200,000 cubic inches into cubic yards.

(311.) If 24 yards of cloth cost 32*l.* how many may be bought for 72*l.*?

(312.) If a creditor to the amount of 45*s.* receive 35*s.*, how much in the 1*l.* is this?

(313.) Reduce 73*l.* 14*s.* 8*d.* to pence; 83765 farthings to guineas; 735 tons, 12 cwt. 3 qrs. to lbs.; 876543 days to years; and 22032 square inches to square yards.

(314.) If 6 lbs. of tea cost 26*s.* how much will 28 lbs. cost?

(315.) If 10 men can do a piece of work in 8 days, how long will 4 men take to do the same?

(316.) If 1*l.* 18*s.* is paid for 19 yards, how many can be purchased for 5*l.* 14*s.*?

(317.) A ship is provisioned for 60 days, allowing each man 3 lbs. a-day: how much should be allowed to make these provisions last 80 days?

(318.) Reduce 147*l.* 9*s.* 4*d.* to pence; 17595 farthings to guineas; 367 tons 16 cwt. 1 qr. 14 lb. to lbs.; 134104 minutes to weeks; and 139968 cubic inches to cubic yards.

(319.) If 12 lbs. cost 2*l.* 12*s.*, how much will 56 lbs. cost?

(320.) If 19*s.* is paid for 9½ yds., how many yards can be purchased for 11*l.* 8*s.*?

(321.) If the penny loaf weigh 4½ oz. when a sack of flour costs 38*s.*, what should it weigh when the sack costs 57*s.*?

(322.) Reduce to a single expression :

$\frac{15}{17} + \frac{5}{34} - \frac{2}{51}$, and $\frac{5}{2} \times \frac{7}{4} \times \frac{1}{15} \times \frac{12}{11} \times 6\frac{2}{7}$,

and also Divide $1l.\ 1s.\ 4\frac{1}{2}d.$ by $\frac{18}{230}$.

(323.) Express

$\frac{2}{7}$ of $21l. - \frac{2}{5}$ of $6s.\ 8d.$ as the fraction of 1 crown; and

$\frac{4}{5}$ of $\frac{1}{2}$ crown $+ \frac{5}{4}$ of $\frac{1}{2}$ guinea as the fraction of 1 guinea.

(324.) How often is $\frac{3}{17}$ of $2s.\ 10d.$ contained in $6l.$?

(325.) If $\frac{3}{7}$ of a prize is worth $3l.\ 12s.$, how much is $\frac{1}{8}$ worth?

(326.) Find the value of

271 yards at $6l.\ 15s.\ 10\frac{3}{4}d.$ per yard; and of

21a. 2r. 12p. at $3l.\ 15s.\ 8d.$ per acre.

(327.) Multiply the 3d power of $\frac{5}{2}$ by the 2d power of $\frac{7}{5}$.

(328.) Required the square roots of 97535376, of 761.9, and of .0003841, both to 6 places of decimals.

(329.) Required the cube roots of 219365327791, and of 11, to 5 places of decimals.

(330.) If the two terms of the fraction $\frac{17}{625}$ be multiplied by a certain number, the fraction will be in the decimal form: what is the number?

(331.) Reduce $\frac{9}{16}$, $\frac{35}{49}$, and $\frac{1}{32}$ to decimal fractions, and .027303 and .5005 to vulgar fractions.

(332.) Perform the following operations :—

1. 17.765 — .93125 and 2020.01 — 199.0245.
2. .0003 × .01 × 500000.
3. 22.5 × .0241 × .0024.
4. .00128 ÷ 8.192.
5. 1708.4592 ÷ .00024.

(333.) A ship's crew consists of 20 men ; the quantity of biscuit will allow 18 oz. to each man per diem ; immediately after weighing anchor, some men are picked up, and the allowance is reduced to 15 oz. : how many were picked up ?

(334.) Reduce to a single expression, $\frac{15}{17} + \frac{5}{34} - \frac{2}{51}$.

(335.) Express $\frac{4}{5}$ of $\frac{1}{2}$ crown + $\frac{5}{4}$ of $\frac{1}{2}$ guinea as the fraction of 1 guinea.

(336.) If the length of the year be taken at $365\frac{1}{4}$ days instead of 365.242264 days, its true value, what will the error amount to in 4 centuries ?

(337.) The edge of a cube is 7 ft. 4 in. long : how many imperial gallons will it hold ? — N.B. 1 imperial gallon = 277.274 cubic inches.

(338.) If a pipe, whose diameter is 1.5 inches, fill a cistern in 5 hours, in what time will another, whose diameter is 3.5 inches, fill the same ?

(339.) The *amount* of 320*l.* 10*s.* at 5 per cent. compound interest, for 4 years ?

(340.) The price of a ton is $96\frac{3}{4}l.$; the commission charged to both seller and buyer is 2*s.* 6*d.* on each ton : how many tons will 775*l.* purchase, and what will the holder receive ?

(341.) By selling at 46*l.* 16*s.* per cwt. there is a loss of 10 per cent. : required the loss or gain per cent. by selling at 48*l.* per cwt.

(342.) 40 gallons, at 2*d.* per gallon,
60 „ 4*d.* „
80 „ 5*d.* „
} are mixed together.

Required the value of 1 gallon.

(343.) How much British money is equivalent to 6012 rupees, exchange being at 2*s.* 1½*d.* per rupee ?

(344.) Extract the cube root of 11 to 5 places of decimals.

(345.) Change .1875 into a vulgar fraction in its lowest terms, and $\frac{17}{2560}$ into a decimal fraction.

(346.) Find—

1. The approximate value of $\frac{5}{13}$.
2. The sum of 3.419 + .0256 + .08 + 1.21734.
3. The excess of 3.025 above .003025.
4. The acres, roods, and poles in 13.3375 acres; and
5. The quotient to 3 places of decimals of
   .0719 + 27.53.

(347.) What length must be cut off from a rectangular surface 2¼ feet broad, to make a square yard ?

(348.) Find the depth of a rectangular cistern whose length = 9¾ feet; breadth = 6 feet; and vol. = 294¼ cubic feet.

(349.) Find the cost of carpeting a room, 28 feet long and 19 feet wide, with carpet ¾ yard wide, at 5*s.* 9*d.* the linear yard.

(350.) Find the cost of 60 panes of glass, each 15 in. by 11¼ in., at 5*s.* 4*d.* the square foot.

(351.) If the 4 per cents. are at 82⅛, how much stock should be given in exchange for an estate which is worth 1750*l.* ?

(352.) If 4 per cent. be lost by selling at 2*s.* 9*d.*, what price will produce a gain of 10 per cent. ?

(353.) By selling at 4*l.* 14*s.* 6*d.*, 18 per cent. is lost: required the prime cost.

(354.) *A*, *B*, and *C*, form a joint stock of 45,000*l.* and gain 15,000*l.*; their respective shares are 7500*l.*, 5000*l.*, and 2500*l.* : find the portion of stock contributed by each.

(355.) *A* advances 1500*l.* for 9 months, and *B* 1200*l.* for 6 months: what share of gain of 1150*l.* belongs to each?

(356.) Ten bushels at 3*s.* 8*d.*; 21 at 3*s.* 10*d.*; 35 at 4*s.* are mixed together: required the price of 1 bushel of this mixture.

(357.) 100*l.* is due in 3 months, 120*l.* in 2 months, and 160*l.* in 5 months: find the equated time.

(358.) Reduce 156*l.* 15*s.* to francs, the exchange being at 23.5 francs per 1*l.*

(359.) If 21*s.* = 27 francs, what should be given for 2500 francs?

(360.) Find the commission on 126*l.* at $\frac{5}{8}$ per cent., and reduce the answer to the decimal of 1*l.* 11*s.* 6*d.*

(361.) Reduce to a single fraction—

$\frac{11}{70} + \frac{5}{21} + \frac{6}{5} + \frac{17}{42}$, and $\frac{1}{2}$ of $\frac{7}{12}$ of $\frac{3}{5} \times \frac{4}{11}$ of $3\frac{1}{7}$.

(362.) Divide 7*s.* 1½*d.* by $\frac{9}{115}$.

(363.) Find the value of—

$$\frac{5}{8} \text{ of } 1l. + \frac{13}{20} \text{ of } 5l. - \frac{2}{5} \text{ of } 6s.\ 8d.$$

(364.) Express as the fraction of 1*l.*—

$$\frac{5}{14} \text{ of a guinea} + 1\tfrac{3}{4} \text{ of a shilling.}$$

(365.) How often is ⅗*l.* contained in 24 guineas?

(366.) If $\frac{3}{16}$ of a ton is worth 4*l.* 10*s.*, what is the value of $\frac{1}{5}$?

(367.) Find the values of 437 cwt. at 3*l.* 6*s.* 9¾*d.*, and of 7 cwt. 3 qrs. 11 lbs. at 2*l.* 13*s.* 1*d.* per quarter.

(368.) *A* has 1385 yards of linen worth 2*s.* 7½*d.* per yard, for which *B* gives him 32*l.* 7*s.* 6*d.* ready money, and also calicos at 10½*d.* per yard. How many yards of the latter did *A* receive?

(369.) 137 cwt. are bought at 1*l.* 14*s.* 9½*d.* per cwt., and sold at 4½*d.* per lb. Required the whole loss or gain.

(370.) *L*, *M*, and *N* continue in trade for a year with a joint stock of 3500*l.* At the end of that time *L*'s share of the gain was 125*l.*—*M*'s 240*l.*—*N*'s 135*l.* Required each partner's share of the stock.

(371.) 1750*l.* is to be paid as follows:

300*l.* in 4 months
125*l.* in 5 months
365*l.* in 8 months
400*l.* in 10 months
The remainder in 12 months

} Required the equated time for the payment of the whole.

(372.) A merchant in London remits to his correspondent in Venice 623*l.* 11*s.* 6*d.* For how many lire will his correspondent draw upon his banker there, when the exchange is at 47*d.* per 6 lire?

(373.) Perform the following operations:—

1. $\frac{11}{70} + \frac{5}{21} + \frac{6}{5} + \frac{17}{42}$.

2. $\frac{1}{2}$ of $\frac{7}{12}$ of $\frac{5}{3} \times \frac{4}{11}$ of $3\frac{1}{7}$.

3. $1.1 \times .011 \times 2.02 \times .002$.

4. $4.234 \div .00365$.

5. Express as the fraction of 1*l.*

$$\frac{5}{14} \text{ of a guinea} + 1\tfrac{3}{4} \text{ of a shilling.}$$

(374.) The edge of a cube of marble is 6 feet; the weight of 288 solid inches is 6 lb.; the value of 1 lb. is sixpence: required the solidity, weight, and value of the block.

(375.) Divide the cube root of $\frac{2460375}{4096}$ by the fourth root of 50625.

(376.) A loss of 10 per cent. is sustained by selling at 46*l.* per ton; what would be lost per cent. by selling at 50*l.* per ton?

(377.) The rates of exchange being 1*l.* = 25.4 francs,
105 kreutzer = 3.75 francs,
60 kreutzer = 1 florin;
and 1 German mile being = $4\frac{1}{2}$ English miles: find the expense (in English money) of travelling 381 English miles, at $1\frac{1}{2}$ florins per German mile.

(378.) At twelve o'clock a watch is set right; at 1 o'clock the watch shows 12h. 55m.; required the correct time when the watch shows 1 o'clock.

(379.) If 5 men, *or* 7 women can perform a piece of work in 35 days, in what time can 7 men *and* 5 women do the same?

(380.) How much stock must be bought at 76*l.* per cent. in order that by selling out at 95*l.*, 95 guineas may be gained?

(381.) *A* delivers to *B* $31\frac{1}{4}$ yards at 5*s.* 6*d.*, and 78 yards at 7*s.* 8*d.*, and receives in exchange wool at 1*s.* 3*d.* per lb.; how many lbs. of wool does he receive?

(382.) 118 lbs. are bought at 3*s.* 10*d.*, and sold at 2*s.* $10\frac{1}{2}$*d.*; how much is lost by the transaction?

(383.) *A*'s stock of 85*l.* was 16 months in trade,
B's stock of 170*l.* was 24 months in trade,
C's stock of 1700*l.* was 5 months in trade,
they gain 270*l.* 13*s.* 6*d.*; required each partner's share of the gain.

(384.) If 200*l.* is due 3 months hence,
If 300*l.* is due 8 months hence,
If 500*l.* is due 12 months hence,
required the equated time of payment.

(385.) If 20 bushels of flour at 1*s.* 10*d.*
42 bushels of flour at 1*s.* 11*d.*
70 bushels of flour at 2*s.* 0*d.*
} are mixed together,
required the value of 1 bushel.

(386.) If six iron bars, each being 4 ft. long, 3 in. broad, 2 in. thick, weigh 288 lbs., how much will 15 weigh, each being 6 ft. 6 in. long, 4 in. broad, 3 in. thick?

(387.) Find the value of

$$5\tfrac{1}{3} - 2\tfrac{5}{6} - 3\tfrac{3}{10} + \tfrac{1}{2}^{3} - 16\tfrac{1}{4} + 3\tfrac{2}{12} + 8\tfrac{1}{3}.$$

(388.) From the following vulgar fractions *select* those which can be exactly expressed by decimal fractions:

$$\frac{3}{12};\ \frac{7}{12};\ \frac{49}{28};\ \frac{47}{28};\ \frac{22}{55};\ \frac{21}{55}.$$

(389.) Express 12 h. 55′ 21″ as the decimal of a day.

(390.) There is a plantation in the form of a hollow square, whose side is 252 yards, the depth is 16 yards; find the area of the plantation, and of the inner square.

(391.) The area of a circle being 3.1416 times the square on the radius; find, to two places of decimals, the length of the tether when a donkey has an acre to feed upon. N.B. 4840 square yards = 1 acre

(392.) Required the *amount* of 350*l.* in 3 years, at 5 per cent. compound interest.

(393.) Required the present worth of 472*l.* 10*s.* due one year hence, at 5 per cent.

(394.) How much per cent. is 4½*d.* on every shilling?

(395.) Reduce 156*l.* 17*s.* into thalers and groschen, it being given that 25.32 francs = 1*l.*, and 15.4 francs = 4 thalers, and 1 thaler = 10 groschen.

(396.) A cubic foot of water weighs 63 lbs.; required the weight of water in a rectangular vessel, whose dimensions are, 1 ft.; 16 ft. 7 in.; 8 ft. 4 in.

(397.) If a tradesman, in *marking* his goods, adds 20 per cent. to the cash price, what will he take for an article which is marked 26*s.*?

(398.) A ladder placed 12 feet from the wall of a house reaches a window 45 feet high; find its length.

(399.) Divide 3⅓ guineas among 6 persons, so that their shares may be in the proportion of the reciprocals of the first 6 units.

(400.) A person, whose annual income consists of 200*l.* arising from property, and 600*l.* from profession, occupies a house at 100*l.* per annum; he pays 7*d.* in the 1*l.* on the WHOLE of his income, and 9*d.* in the 1*l.* on the rent of his house; but if the tax on the 600*l.* be reduced to 5¼*d.*, and the house-tax doubled, how would he be affected by the change?

ANSWERS

TO THE

QUESTIONS ON ARITHMETIC.

ANSWERS

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QUESTIONS ON ARITHMETIC.

FRACTIONS.

Ex.	Ans.
2. . . .	$\frac{14}{7}$; $\frac{35}{7}$; $\frac{56}{7}$
3. . . .	$\frac{165}{11}$; $\frac{891}{11}$
4. . . .	$\frac{378}{21}$; $\frac{672}{21}$;
	$\frac{861}{21}$; $\frac{2352}{21}$
7. . . .	$\frac{35}{6}$; $\frac{134}{9}$; $\frac{263}{12}$
8. . . .	$\frac{113}{3}$; $\frac{160}{9}$; $\frac{207}{7}$
9. . . .	$\frac{2881}{267}$; $\frac{1799}{16}$
10. . . .	$\frac{149}{12}$; $\frac{5029}{16}$
11. . . .	$\frac{6130}{17}$; $\frac{8058}{1974}$
12. . . .	$\frac{42985}{111}$; $\frac{226847}{234}$

Ex.	Ans.
15. . . .	$15\frac{2}{5}$; $13\frac{9}{17}$
16. . . .	$182\frac{5}{21}$; $57\frac{13}{22}$
17. . . .	$99\frac{13}{39}$; $9\frac{111}{234}$
18. .	$357\frac{20}{20}$; $2019\frac{30}{2019}$
21. . . .	$\frac{1}{8}$; $\frac{7}{12}$; $\frac{8}{55}$
22. . .	$\frac{35}{144}$; $\frac{39}{220}$; $9\frac{17}{108}$
23. . . .	$\frac{1}{9}$; $10\frac{5}{16}$
25. . . .	16; 192
26. . . .	15; 192
27. . . .	36; 128
28. . . .	587; 2223
29. . . .	113; 47
30. . . .	85; 17
32. . . .	16; 192;
	15; 192
33. . . .	36; 128;
	587; 2223

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FRACTIONS.

Ex.	Ans.
2....	$\frac{14}{7}$; $\frac{35}{7}$; $\frac{56}{7}$
3....	$\frac{165}{11}$; $\frac{891}{11}$
4....	$\frac{378}{21}$; $\frac{672}{21}$;
	$\frac{861}{21}$; $\frac{2352}{21}$
7....	$\frac{35}{6}$; $\frac{134}{9}$; $\frac{263}{12}$
8....	$\frac{113}{3}$; $\frac{160}{9}$; $\frac{207}{7}$
9....	$\frac{2881}{267}$; $\frac{1799}{16}$
10....	$\frac{149}{12}$; $\frac{5029}{16}$
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Ex.	Ans.
15....	$15\frac{2}{5}$; $13\frac{2}{17}$
16....	$182\frac{5}{21}$; $57\frac{13}{22}$
17....	$99\frac{13}{39}$; $9\frac{111}{234}$
18..	$357\frac{20}{209}$; $2019\frac{30}{2019}$
21....	$\frac{1}{8}$; $\frac{7}{12}$; $\frac{8}{55}$
22...	$\frac{35}{144}$; $\frac{39}{220}$; $9\frac{17}{108}$
23....	$\frac{1}{9}$; $10\frac{5}{16}$
25....	16; 192
26....	15; 192
27....	36; 128
28....	587; 2223
29....	113; 47
30....	85; 17
32....	16; 192;
	15; 192
33....	36; 128;
	587; 2223

Ex.	Ans.
34.	113; 47; 85; 17
35.	12; 3; 587; 4199
37.	72
38.	72
39.	6300
40.	252
41.	1824
42.	2520
44.	778050
45.	3586451
46.	1120694
48.	$\frac{3}{4}$; $\frac{7}{8}$; $\frac{3}{7}$; $\frac{5}{6}$
49.	$\frac{8}{9}$; $\frac{5}{6}$; $\frac{9}{14}$; $\frac{12}{13}$
50.	$\frac{7}{8}$; $\frac{7}{13}$; $\frac{1}{3}$; $1\frac{7}{15}$
51.	$\frac{1}{15}$; $\frac{1}{7}$; $\frac{343}{750}$; $\frac{1}{3}$
52.	$\frac{32}{77}$; $1\frac{3}{79}$; $\frac{1709}{84229}$
53.	$\frac{121}{456}$; $6\frac{86}{113}$; $\frac{50}{161}$
54.	$\frac{22}{999}$; $\frac{7}{640}$
55.	$\frac{13}{45}$; $\frac{19}{34}$
56.	$\frac{113}{355}$; $\frac{22}{999}$
59.	$\frac{10}{12}$; $\frac{7}{12}$

Ex.	Ans.
60.	$\frac{8}{12}$; $\frac{9}{12}$
61.	$\frac{12}{27}$; $\frac{5}{27}$
62.	$\frac{16}{40}$; $\frac{25}{40}$
63.	$\frac{105}{210}$; $\frac{70}{210}$; $\frac{42}{210}$; $\frac{30}{210}$
64.	$\frac{12}{48}$; $\frac{18}{48}$; $\frac{16}{48}$; $\frac{15}{48}$
65.	$\frac{168}{456}$; $\frac{40}{456}$; $\frac{209}{456}$
66.	$\frac{140}{1932}$; $\frac{189}{1932}$; $\frac{132}{1932}$
67.	$\frac{350}{525}$; $\frac{315}{525}$; $\frac{375}{525}$; $\frac{126}{525}$
68.	$\frac{700}{1680}$; $\frac{315}{1680}$; $\frac{80}{1680}$; $\frac{196}{1680}$
69.	$\frac{900}{1260}$; $\frac{1120}{1260}$; $\frac{1155}{1260}$; $\frac{1092}{1260}$; $\frac{1190}{1260}$

Ex.	Ans.
70.	$\frac{800}{760}$; $\frac{798}{760}$; $\frac{665}{760}$; $\frac{88}{760}$
75.	$\frac{1}{288}$
76.	$\frac{4}{147}$
77.	$\frac{5}{32}$
78.	$\frac{825}{2304}$
79.	$\frac{20}{27}$; $\frac{20}{147}$
80.	$\frac{3}{44}$; $\frac{2}{75}$
81.	$\frac{145}{378}$
82.	$4\frac{43}{110}$
83.	$\frac{35}{96}$
84.	$\frac{3}{56}$; $\frac{959}{2680}$
85.	$\frac{1}{280}$
86.	$\frac{1}{840}$
87.	$5\frac{114}{432}$
88.	$\frac{251}{968}$
89.	$\frac{10}{27}$
90.	$\frac{1}{28}$

Ex.	Ans.
91.	$\frac{5}{884}$
92.	$\frac{107}{105}$
93.	$\frac{137}{144}$
94.	$\frac{1908}{343375} = \frac{1}{180}$ nearly.
95.	$\frac{1431}{84211}$

Ex.	£	s.	d.		s.	d.
99.		3	4	;		$7\frac{1}{2}$
100.	1	2	6	;	17	6
101.		17	4	;	11	3
102.		6	8	;	3	$8\frac{3}{4}$
103.		18	$5\frac{1}{10}$	;	7	3

Ex.	oz.	dwt.		qrs.	lbs.
104.	7	4	;	2	20

Ex.	hrs.	min.		dys.	hrs.	min.
105.	7	12	;	2	3	27

Ex.	yds.	ft.	in.		yds.
106.	82	1	6	;	60

Ex.	bush.	pk.	gall.	pks.	gall.	qt.	pt.
107.	1	1	1	;	2	1	1	$1\frac{3}{7}$

Ex.	qrs.	na.	in.	
108.	2	3	$\frac{1}{4}$	;

mil.	fur.	po.	yd.	ft.	in.
1	1	13	1	2	6

Ex.	oz.	dwt.	gr.		oz.	dr.
109.	6	10	$11\frac{2}{61}$	;	9	$2\frac{2}{7}$

Ex.	qrs.	lbs.
110.	2	16

cwt.	qrs.	lbs.	oz.	dwt.
8	3	15	8	$14\frac{2}{5}$

Ex.	Ans.					
	oz.	dr.	scr.	wks.	dys.	hrs.
111.	2	2	2;	2	5	18
113.			$\frac{11}{48}$;			$\frac{7}{55}$
114.			$\frac{153}{320}$;			$\frac{101}{252}$
115.			$\frac{267}{560}$;			$\frac{663}{35840}$
116.			$\frac{871}{3600}$;			$\frac{929}{2016}$
117.				$\frac{3}{5}$;		$\frac{11}{25}$
118.						$\frac{25}{264}$
123.						$1\frac{1}{30}$
124.						$\frac{71}{105}$
125.						$2\frac{43}{60}$
126.						$\frac{109}{483}$
127.						$\frac{173}{210}$
128.						$\frac{239}{323}$
129.						$2\frac{68755}{74046}$
130.						$31\frac{9}{35}$
131.						24
132.						$5\frac{43}{312}$
133.					s. 2	d. $\frac{1}{105}$
134.					16	$\frac{1}{2}$

Ex.	Ans.			
	qrs.	lbs.	oz.	dr.
135.	2	11	0	$7\frac{1}{3}$
	wks.	dys.	hrs.	min.
136.	7	6	18	48
			£ s.	d.
137.			1 9	$\frac{3}{5}$
140.			$\frac{5}{24}$;	$\frac{5}{527}$
141.			$2\frac{39}{35}$;	$5\frac{3}{4}$
142.			$17\frac{179}{364}$;	$3\frac{13}{15}$
143.			£ s. 0 11	d. $8\frac{1}{40}$
144.	$\frac{1}{380}$ of a shilling.			
				dwt. gr.
145.				11 9
146.				$1\frac{1}{2}d.$
147.	$\frac{1}{22}$ of a guinea; diff. $= \frac{12}{77}$ of $1d.$			
				fur. yds.
148.				4 88
	cwt.	qrs.	lbs.	oz. dwt.
149.	7	0	27	3 $10\frac{18}{35}$
152.			$\frac{1}{20}$;	$\frac{1}{3}$
153.			$\frac{21}{25}$;	$1\frac{1}{9}$
154.			$\frac{2}{153}$;	$7\frac{1}{12}$
155.				$5\frac{217}{640}$

Ex.	Ans.
156. . . .	1
157. . . .	$\frac{7}{324}$
158. . . .	2
159. . . .	528
	£ s. d.
160. . . .	73 1 $6\frac{3}{4}$
161. . . .	5078 14 $8\frac{1}{24}$
	ton. cwt. qr. lbs.
162. . . .	42 4 1 $19\frac{11}{20}$
166. . . .	$\frac{1}{4}$; $\frac{1}{3}$
167. . . .	$16\frac{17}{32}$; $\frac{1}{140}$
168. . . .	14; 14
169. . . .	$\frac{9}{10}$; $\frac{7}{15}$
	£ s. d.
170. . . .	89 4 $1\frac{1}{23}$
171. . . .	1
	mil. fur. yds.
172. . . .	7 0 124
	cwt. qrs. lbs.
173. . . .	2 2 $16\frac{11}{37}$
174. . . .	$12\frac{125}{16637}$

Ex.	Ans.
	s.
175. . . .	$\frac{175}{243}$
176. . . .	$1\frac{1}{5}d.$
177. . . .	$1\frac{23}{25}s.$
	s. d. f.
179. . . .	9 $8\frac{1}{2}$ $\frac{2}{3}$
	£ s. d.
180. . . .	227 12 1
	d. f. d. f.
181. . .	$2\frac{3}{4}$ $\frac{10}{775}$; $1\frac{3}{4}$ $\frac{431}{775}$
	£ s. d.
182. . . .	320 16 8
	s. d. f.
183. . . .	4 $11\frac{1}{2}$ $\frac{34}{107}$
	yd. qr.
184. . . .	2 $3\frac{18}{21}$
185. . . .	$4\frac{1}{50}$ oz.
186. . . .	$65\frac{13}{14}$ days.
	£ s. d.
187. . . .	3090 18 $2\frac{2}{11}$
188. . . .	75 4 $8\frac{1}{10}$
189. . . .	9 2 0
190. . . .	7 15 $10\frac{3}{5}$

DECIMAL FRACTIONS.

2. . . .	95	6. . . .	16.656055
3. . . .	183.314	8. . . .	24.394
4. . . .	47.2623	9. . . .	5780.2433
5. . . .	1148.87371	10. . . .	270.4946

Ex.	Ans.
11. . . .	3.60126
12. . . .	99.999999
15. . . .	195.099261
16. . . .	175.14225
17. . . .	.00059563
18. . . .	102.030201
19. . . .	.00040208
20. . . .	.0018615608
21. . . .	.002093663
22. . . .	.012024012
30. . . .	.525 ; .62083
31. . . .	.89375 ; .228125
32.	.9947916 ; .8697916
34. . . .	.952380
35.	.10714285 ; .3571428
36.	.6517857142 ; .4375
37. . . .	. . .6875
38. . . .	.123816287
39. . . .	.7583
40. . . .	.167123287
41. . . .	727
42. . . .	.8125
43. . . .	.75
44. . . .	.242
45. . . .	.432142857
46. . . .	. . .66875
48. . . .	d. s. d. 7½ ; 16 . . 7.7496

Ex.	Ans.
49. . . .	qrs. lbs. oz. dr. 2 19 5 1.92 ;
	cwt. qr. 9 2
50. . . .	s. d. 19 1.44 ;
	s. d. 5 9.40104
51. . . .	fur. po. yd. ft. 6 5 4 .54 ;
	mil. fur. po. yd. ft. 1 3 13 0 1.98
52. . . .	s. d. 19 1.95 ;
	£ s. d. 1 17 6
53. . . .	s. d. 9 6.798216 ;
	£ s. d. 2 15 8.016
54. . . .	dys. hrs. min. sec. 3 7 56 3.84 ;
	hrs. min. sec. 16 26 58.56
55. . . .	roo. po. yd. ft. 1 37 9 3.942
	yd. ft. in. 3 1 4.4232
63. . . .	$\frac{13}{99}$
64. . . .	$\frac{14}{33}$
65. . . .	$\frac{1}{11}$

Ex.	Ans.
66. . . .	107 / 333
67. . . .	481 / 3367
68. . . .	30427 / 111111
69. . . .	8 / 15
70. . . .	43 / 90
71. . . .	1697 / 3300
72. . . .	34213 / 49950
73. . . .	$4\frac{53}{1650}$

Ex.	Ans.
74. . . .	$17\frac{687907}{2497750}$
75. . . .	s. d. : 14 $4\frac{8}{15}$; 6 $9\frac{5}{11}$
76. . . .	qr. lbs. oz. : 1 18 $4\frac{8316}{9999}$; oz. dwt. gr. : 5 9 $2\frac{2}{11}$
79. . . .	7726.444 &c.
80. . . .	39965.9978 &c.
81. . . .	3085.40231 &c.
82. . . .	.2708 &c.
84. . . .	54.6587 &c.
85. . . .	164.086 &c.
86. . . .	30.7283 &c.
87. . . .	11.01292 &c.
88. . . .	6.2290190

INTEREST, &c.

	£	s.	d.
2. . . .	49	11	$2\frac{2}{5}$;
	875	11	$2\frac{2}{5}$
3. . . .	365	2	0 ;
	2190	12	0
4. . . .	6	5	$7\frac{1}{2}$;
	131	18	$1\frac{1}{2}$
5. . . .	£1360 ; £3360		
	£	s.	d.
6. . . .	93	17	$9\frac{3}{40}$;
	719	16	$1\frac{23}{40}$
7. . . .	57	7	$4\frac{1}{6}$;
	330	10	$10\frac{5}{8}$

	£	s.	d.
9. . . .	165	4	$4\frac{1}{5}$;
	2000	19	$4\frac{1}{5}$
10. . . .	23	18	$1\frac{29}{40}$;
	297	2	$7\frac{29}{40}$
11. . . .	427	15	$6\frac{24}{25}$;
	2156	3	$6\frac{24}{25}$
12. . . .	81	0	$8\frac{5}{8}$;
	698	9	$0\frac{5}{8}$
13. . . .	2	1	$1\frac{4}{5}$;
	207	16	$1\frac{4}{5}$
14. . . .	64	10	$0\frac{789}{800}$;
	384	0	$6\frac{789}{800}$

Ex.	Ans. £	s.	d.
15....	195	17	$8\frac{7}{20}$;
	908	3	$10\frac{7}{20}$
16....	147	1	$10\frac{10}{80}$;
	972	17	$6\frac{10}{80}$
17....	241	0	$10\frac{347}{400}$;
	1290	19	$4\frac{347}{400}$
18....	252	10	$2\frac{1173}{1600}$;
	2622	2	$4\frac{1173}{1600}$
19....	7	4	$6\frac{22}{25}$;
	730	1	$10\frac{22}{25}$
20....	58	13	$10\frac{330}{400}$;
	4327	8	$8\frac{330}{400}$
21....	104	2	$7\frac{307}{320}$;
	1029	15	$2\frac{227}{320}$
22....	197	8	$6\frac{153}{640}$;
	923	11	$9\frac{473}{640}$
25....	26	14	8
26....	152	4	$0\frac{19}{20}$
27....	286	2	$9\frac{291}{365}$
28....	29	3	4
29....	7	2	$3\frac{3}{4}$
30....	148	1	$4\frac{52}{365}$
31....	12	8	$3\frac{33}{73}$
32....	10	2	$3\frac{57}{73}$
37....	56	5	0
38....	144	11	$6\frac{66}{83}$
39....	29	10	$4\frac{2}{7}$
40....	55	6	$11\frac{319}{427}$
41....	677	17	$9\frac{81}{163}$

Ex.	Ans. £	s.	d.
42....	8	18	$6\frac{378}{2941}$
43....	105	2	$6\frac{198}{365}$
	days = 43		
44....	1243	4	$8\frac{1528}{1825}$
	days = 53		
45....	347	4	$9\frac{15}{73}$
	days = 72		
46....	273	7	$6\frac{754}{1825}$
	days = 84		
47....	490	19	$1\frac{5323}{91125}$
	days = 104		
48....	4202	15	$9\frac{13163}{18225}$
	days = 115		
50....	530	12	$9\frac{1}{4}$
			nearly.
51....	628894	12	$6\frac{1}{4}$
			nearly.
52....	63	0	$6\frac{1}{4}$
			nearly.
57....	5	1	$0\frac{1}{2}$
58....	144	3	0
59....	23	16	$11\frac{29}{200}$
60....	110	8	9
61....	4	10	0
62....	6	11	3
63....	781	5	0
65....	90	9	$0\frac{108}{199}$;
	The price *not* including Brokerage.		
66....	80	1	$4\frac{16}{599}$;
	The price including Brokerage.		

Ex.	Ans. £ s. d.
67. . . .	2788 16 $10\frac{178}{251}$;
	The price including Brokerage.
68. . . .	139 8 $10\frac{1}{10}$ nearly;
	The price including Brokerage.

Ex.	Ans. £ s. d.
69. . . .	3 10 10 per annum;
	The price including Brokerage.
70. . . .	95 11 0;
	The price including Brokerage.

MISCELLANEOUS.

	£ s. d.
71. . . .	483 17 $5\frac{1}{31}$
72. . . .	920 6 $11\frac{3}{4}$
73. . . .	24390 4 $10\frac{22}{41}$
74. . . .	£13125;
	£56 5 0
75. . . .	£6129; $8\frac{968}{6129}$

	£ s. d.
76. . . .	225 13 $11\frac{61}{73}$;
	£ reckoning, dys.
	1000 bearing interest for 83
	700 " " 91
	450 " " 99
	200 " " 86
77. . . .	£1043 $6\frac{158}{1837}$;
	48 days, rate 5 per cent.
78. . . .	£$85\frac{5}{7}$

EQUATION OF PAYMENTS, &c.

2. . . .	$14\frac{2}{3}$ month.
3. . . .	$4\frac{6}{7}$ month.
	mo. wk. dys.
4. . . .	4 3 3 nearly.
5. . . .	$6\frac{1}{2}$
6. . . .	$11\frac{5}{14}$
7. . . .	6 month.
8. . . .	$2\frac{3}{4}$ month.
13. . . .	9s. 4d.
14. . . .	$12\frac{1}{2}$
15. . . .	$10\frac{5}{12}$
16. . . .	$8\frac{1}{3}$

17. . . .	$7\frac{9}{13}$
18. . . .	$42\frac{2}{5}$
19. . . .	$7\frac{1}{3}$
	£ s. d.
20. . . .	0 8 0
21. . . .	2 0 0
22. . . .	0 7 8
23. . . .	$6\frac{21}{160}$
24. . . .	£525
25. . . .	5s. 6d.;
	$26\frac{14}{61}$ per cent.

Ex.	Ans.
6....	$\frac{24}{25}$
7....	16*s.* 8.0804064*d.*
8....	203975; 258
9....	24 days.
	£ *s.* *d.*
10....	22 17 $7\frac{2}{25}$
11....	20 11 $9\frac{9}{31}$
12....	129 13 $6\frac{45654}{390625}$
13....	0 2 $9\frac{3}{5}$ gain.
14....	41 10 $6\frac{4}{7}$
15....	.39479 &c.; .120000
16....	3*s.* $3\frac{3}{7}$*d.*
17....	£7 0*s.* $10\frac{1}{3}\frac{3}{2}$*d.*
18....	Six thousand and thirty; Six hundred and four thousand and fifty-two; Nine hundred millions, three hundred thousand, eight hundred and five; 7007977; 1010200000; or 1000010200000
19....	218289976; 18065364
20....	410724907; .1011924
21....	12588145548; 640131605776000
22....	$9847219\frac{1433891}{3843365}$; $34491\frac{2}{27}$
23....	335675 farthings; 8613 sixpences; 22253 grs.; 30126 perches; 21 guineas;
	qrs. bus. pk. gall. qt. pt.
	2936 7 1 0 0 1;
	1260 shillings; 252 crowns; £63
24....	£4130 6*s.* $8\frac{3}{4}$*d.*
25....	5 cwt. 1 qr. 11 lb.; 1 tn. 5 hhd. 21 gall. 4 pts. if 4 hhd. = 1 tun.
26..	2141 ac. 2 ro. 17 po.; £440 11*s.* $6\frac{3}{4}$*d.*
27...	10*s.* $0\frac{1}{4}$*d.* $\frac{2}{1}\frac{3}{3}\frac{3}{6}\frac{3}{1}$*f.*; 7 mi. 250 yd. 0 ft. 5 in.
	£ *s.* *d.*
28....	4 11 0
29....	1 1 5
30....	4*s.* 4*d.*; 66 Nos.; $31\frac{1}{2}$ Parts.
31....	126 = G.C.M.; 1134 = L.C.M.
32....	$\frac{38576}{103}$; $1658\frac{57}{35531}$;

Ex.	Ans.
	$\frac{2223}{2717}$; $\frac{1463}{2717}$; $\frac{1716}{2717}$;
	$\frac{6}{12}$; $\frac{8}{12}$; $\frac{10}{12}$; $\frac{7}{12}$; $\frac{1}{4}$
33.	$5\frac{1}{3}$; $3\frac{8}{35}$; $2\frac{35}{87}$;
34.	$\frac{2308}{11}$ / $\frac{}{60}$
35.	$\frac{1008}{5} = 201\frac{3}{5}$;
	$\frac{3}{10}$; $\frac{829}{44800}$
36.	£1 6*s*. $\frac{6}{7}$*d*.
37.	$42\frac{61}{192}$ yards.
38.	£1818 0*s*. $0\frac{1857}{2240}$*d*.
39.	$\frac{13}{16}$; $\frac{7}{800}$;
	$\frac{1727}{125} = 13\frac{102}{125}$
40.	401.02486;
	32.58133;
	126029.501025;
	10; 689.655 &c.
41.	.04; $\frac{265}{999}$
42.	.5625; 2gall. 1qt.
43.	£41 4*s*. $11\frac{1}{4}$*d*.;
	£675 6*s*. 5.7449376*d*.

Ex.	£	*s*.	*d*.
44.	398	7	8;
	22102	7	$6\frac{1}{2}$
45.	0	8	$6\frac{1}{4}$

Ex.	Ans.
46.	64640

Ex.	£	*s*.	*d*.
47.	421	8	$10\frac{1}{2}$
48.	1067	18	$4\frac{1}{2}$
49.	2057	1	4

Ex.	Ans.
50.	$4\frac{1}{2}$ per cent.
51.	7*s*. 8*d*.
52.	5*s*. 6*d*. per lb.;
	$26\frac{14}{61}$ per cent.
53.	£276.83437 &c.
54.	1*s*. 7*d*.
55.	£3 15*s*.
56.	1234; 175.25
57.	60 feet.
58.	345
59.	$4969\frac{49}{54}$ cub. ft.;
	or 4969 10′ 10″ 8‴;
	£7 15*s*. $3\frac{103}{144}$*d*.
60.	$\frac{2}{3}$; $\frac{9}{16}$
61.	12*s*. $7\frac{1}{2}$*d*.
62.	.9125; 8*s*. $11\frac{1}{10}$*d*.
63.	144 men.
64.	$\frac{32}{243}$
65.	1.124864
66.	1728
67.	90101
68.	48 ft.
69.	2.3513 &c.
70.	39.25 yds. nearly.

Ex.	Ans. £	s.	d.	f.
71.	33	3	$4\frac{1}{4}$	$+\frac{1}{32}$
72.	741	0	0	
73.	591	12	4	
74.	33	3	$4\frac{1}{4}$	$+\frac{1}{32}$
75.	2	12	6	
76.	7	13	$11\frac{1}{4}$	$+\frac{21}{25}$
77.	5	17	1	$+\frac{114}{125}$
78.	1925	6	$6\frac{1}{4}$;	
	1906	8	9;	
	10552	15	9	
79.	1315	16	$11\frac{1}{2}$	$+\frac{1}{2}$
	76	9	6	

80. £37 2s. 6d. reckoning 3 mo. 15 dys. = 14 wks. 1 dy.

	£	s.	d.	f.
81.	15	6	3	
82.	5	10	$2\frac{1}{4}$	$+\frac{41}{191}$

83. $1\frac{7}{60}$; $\frac{71}{105}$

	£	s.	d.
84.	0	7	$7\frac{3}{4}$;
	2	16	3

85. 1 cwt. 2 qrs. 21 lbs.

86. $\frac{379}{480}$; $\frac{4621}{17532}$

87. $\frac{2800}{2660}$; $\frac{2793}{2660}$; $\frac{3040}{2660}$

fractions of 1s.

88. $\frac{5}{39}$; .405

Ex.	Ans.
89.	$1\frac{111}{104}$; 1200; .12
90.	10.03322 &c.
91.	11s.; .0055
92.	£446 8s. $9\frac{15}{73}$d.
93.	27
94.	909; 1.01017 &c.
95.	2000; .0005

	£	s.	d.
96.	3	15	2
97.	467	1	$6\frac{6}{7}$
98.			$585\frac{5}{9}$

99. .02916; 38227, &c.

	s.	d.	
100.	4	$6\frac{9}{50}$;	.240625

	£	s.	d.
101.	783	3	0

102. $\frac{55}{64}$; 109; 7.2

	£	s.	d.
103.	192	18	9

104. 23 = G. C. M.

105. 908; 1.015155 &c.

	£	s.	d.
106.	5	11	$5\frac{43}{61}$
107.	15	11	$8\frac{11}{12}$
108.	160	9	$7\frac{1}{5}$

109. $1\frac{11}{12}$; $\frac{15}{26}$; $\frac{69}{73}$

110. .007257; 120000

111. 2.2360; 2.7494

112. $5\frac{55}{66}$

113. $7\frac{1}{5}$ mo.

114. .91875; $.\dot{7}$

Ex. Ans.

s. d. f.

115. . . . 1.44;
17 $6\frac{1}{2}$ + .9952;
7 $10\frac{1}{2}$

116. . . . 92 ft. 2′ 5″;
ft. in.
or, 92 29

£ s. d. f.

117. . . . 230 1 $5\frac{1}{4}$ + $\frac{8}{25}$

118. . . . 6561; 512;
29.6063 &c.

119. . . . $2\frac{2}{3}$ days.

£ s. d.

120. . . . 11 5 $1\frac{7}{8}$

121. . . . 1236 18 9.7725

122. . . . $\frac{13}{16}$; $\frac{13919}{499500}$

123. . . . 5168$\frac{4}{7}$ guineas;

lbs. oz. dwt. gr.

50 9 10 11

dys. hrs. min. sec.

5 16 27 36

£ s. d.

124. . . . 5483 2 $2\frac{1}{4}$;

tons. cwt. qr. lbs.

54 6 1 20

£ s. d.

125. . . . 14 7 $6\frac{4}{97}$; 82

126. . . . 176 10 0

127. . . . 1 $1\frac{1}{2}$

128. . 1080 0 0.

129. . 36044 3 4.;
2266 8 $8\frac{1}{12}$

Ex. Ans.

£ s. d. f.

130. 13 $5\frac{1}{2}$ + .5860928;
1 18 $6\frac{3}{4}$ + .2150976

131. 23 14 $\frac{1}{2}$ + .6958816
&c.
for 199 days.

132. . . . 7906; 945

133. . 1.69999978867216
= 1.3038404 × 1.3038404;
1.30384048

134. . . . $4\frac{7}{8}$ yards.

135. . . . £262.96

136. . . . $3\frac{31}{75}$ days.

137. . . . 102.3466 &c.

138. . . . .621459 &c.

£ s. d.

139. . . . 2 2 $11\frac{65}{72}$

140. . . . £
Interest = 21.89808 nearly;
= 21 17 11.5 &c.

£
Amount = 153.14808
nearly;

£ s. d.
or 153 2 11.5 &c.

141. 1.1; 3.47850543 &c.;
439

oz. lbs. oz.

142. . . . $42\frac{7}{54}$ = 2 $10\frac{7}{54}$
avoird. wt.

£ £ s. d.

143. . . . 240; 2026 13 4

144. 22

Ex.	Ans.
145. . . .	80
	£ s. d.
146. . . .	33 6 8
147. . . .	4 6 $7\frac{1}{2}$
148. . . .	$4\frac{8}{25}$ days.
	gall. qt.
149. . . .	30 1
	£ s. d.
150. . . .	14 1 3
151. . . .	324.612 &c.
152. . . .	. . . $2\frac{3}{10}$
153. . . .	Sum $= \frac{128}{15}$;
	diff. $= \frac{32}{15}$;
	and 128 = 4 times 32
154. . . .	. . . 133 / 540
155. . . .	$14\frac{7}{15}$
156. . . .	$10\frac{1}{2}$; $\frac{1}{3136}$
157. . . .	$\frac{16}{24}$; $\frac{18}{24}$; $\frac{21}{24}$
158. . . .	.16; .72
159. . . .	745.7961
160. . . .	20796.875
161.	.628125; .790625
	£ s. d.
162. . . .	2 7 $4\frac{1}{2}$;
	ac. ro. po.
	13 1 20
163. . . .	999; 49

Ex.	Ans.
	£ s.
164. . . .	7 6.45883
165. . . .	14 men.
	ft. in.
166. . . .	200 $5\frac{5}{11}$
	£ s. d.
167. . . .	5 12 6;
	6 11 3;
	7 10 0;
	£ s. d.
	8 8 9;
	9 7 6;
	11 5 0
168. . . .	£45 = A's;
£90 = B's;	£225 = C's.
	£ s. d.
169. . . .	400 14 $3\frac{3}{7}$
170. . . .	$108\frac{7}{25}$ planks.
	£ s. d.
171. . . .	578 16 3
172.	$\frac{352}{440}$; $\frac{1155}{440}$; $\frac{1400}{440}$
173. . . .	$14\frac{17}{1320}$
174. . . .	$39\frac{13}{648}$
175. . . .	133 / 540
176. . . .	$19\frac{87}{95}$
177. . . .	. . 113 / 123
	£ s. d.
178. . . .	1 4 0
179. . . .	6666 6 8
180. . . .	$23\frac{16}{81}$

Ex.	Ans.
181. . . .	$5\frac{9}{14}$ mo.
182.	£437.8787683 nearly.
183. . . .	$34\frac{2}{5}$ per cwt.
184. . . .	*A* receives 3818; *B* receives 5727; *C* receives 9545
185. . . .	4.79
186. . . .	36
	£ s. d.
187. . . .	3 1 5
188. . . .	3605 fr.
	ton. cwt. qrs. lbs. oz.
189. . .	4 4 3 23 6;
	mi. fur. yd. ft. in.
	1 1 73 2 6;
	.49395;
	.782512
	£ s. d.
190. . . .	3012 3 7
191. . . .	8620 1 $10\frac{1}{2}$
192. . . .	72 men.
193. . . .	572
	cwt. qrs. lbs.
194. . . .	24 3 23
195. . . .	54 3 5
196. . . .	14 3 $19\frac{9}{32}$
	deducting for tare first.
	£ s. d.
197. . .	377 3 $0\frac{1}{4}$.
198. . .	584 3 $10\frac{3}{4}$ nearly.
199. . .	2 13 $7\frac{3}{4}\frac{353}{1000}$
200. . .	2 16 $11\frac{1}{4}\frac{1551}{1825}$
201. . .	237 10 0

Ex.	Ans.
	cwt. qrs. lbs.
202. . . .	94 2 $16\frac{25}{26}$
	deducting in the given order.
	£ s. d.
203. . . .	28 15 $4\frac{1}{2} + \frac{9}{25}$
204. . . .	64 11 8
205. . . .	24 1 4 + $\frac{93}{100}$
206. . .	1600 0 0
	cwt. qrs. lbs.
207. . . .	30. 2 $11\frac{1}{5}$
208. . . .	£2; 250 cwt.
	£. s. d.
209. . . .	28 12 0
	lbs. oz.
210. . . .	2 $12\frac{9}{14}$
	£ s. d.
211. . . .	1 6 $1\frac{3}{7}$
212. . . .	13 / 18
213. .	.001300492417455
	ft. ft. in.
214. 100	7′= 100 84;
59	7′= 59 1008
215. . . .	2.236068
216. . . .	33
217. . . .	$1\frac{1}{2}$ year.
218. . . .	15647.374 fr.
	s. d. f.
219. . . .	10 $2\frac{1}{2} + \frac{1}{7}$
220. . . .	$43\frac{1}{6}\frac{1}{9}$ lbs.
221. . .	273.642102 miles.
	£ £
222.	*A*'s=540; *B*'s=720; *C*'s=1050; *D*'s=1290

Ex.	Ans.
223. . . .	$10\frac{1}{30}d.$
224. . . .	14 mo.
225. . . .	$1874\frac{18}{43}$ lbs.
226. . . .	4
227. . . .	A's = £175; B's = £336; C's = £189
228. .	$4\frac{1}{2}$ mo. = 138 days.
229. . . .	£ 192 s. $0\frac{8}{19}$
230. . . .	marks. 7757 sch. 8
231. . . .	£ 4 s. 2 d. $7\frac{80}{121}$;
	4 19 $0\frac{12}{101}$;
	4 11 $4\frac{56}{739}$;
	5 9 $4\frac{44}{463}$
232. . . .	52 10 0;
	4 5 0;
	31 5 0;
233. . .	.5625; .714285; $\frac{9101}{1000000}$; $\frac{1001}{10000}$
234. . . .	16.833639;
	100.9981;
	.000048884
	1160
235. . . .	£ 125 s. 13 d. $10\frac{1}{4}$;
	.127109375
236. . . .	3956 miles.
237. . . .	79 ft. 8′ 8″;
	or 79 ft. 104 in.

Ex.	Ans.
238.	49598 ft. 8′ 7″ 6‴
	or 49598 ft. 1242 in.
239. . .	$\frac{12}{15} = \frac{4}{5} = \frac{8}{10} = .8$
240. . . .	$\frac{3}{3200}$
241. . . .	2492.2622123;
	3.44902; 15
	.0001234321;
	1568.6274 &c.;
	$\frac{14}{165}$
242. . . .	£ 0 s. 7 d. $10\frac{1}{2}$
243. . . .	8 1 6.75
244. . . .	56 18 $4\frac{3}{4}$;
	412 13 $4\frac{5}{4}$
245. . . .	0 9 0
246. . . .	£ 296 s. 15 d. $11\frac{1}{2} + \frac{1}{2}$
247. . . .	7 16 $6\frac{1}{2}$
248. . . .	0 7 0
249. . . .	cwt. 27 qrs. 2 lbs. $16\frac{2}{13}$
	deducting in the given order.
250. . . .	48 2 $4\frac{103}{182}$
	deducting in the given order.
251. . . .	$1\frac{1}{2}$ year.
252. . . .	.000125 cub. ft.
	= .216 cub in.
253. . . .	.0000000001;
	$\frac{16}{625}$

Ex. Ans.

254. . . . 847 ; 2.236068 ;
49
121

255. . . . $\frac{3}{2}$; 2.884499

256. . . . 33

257. . . . 585.311028

258. . . . 58.43515 ;
100.011 ;
.001300492417455 ;
.08339

259. . . . .006993 ; .3125

	hrs.	min.	sec.
260. . . .	5	48	50.9184

261. . . . 401 ft. 6′ 4″ ;
or 401 ft. 76 in.

262. . . . 100 ft. 7′ ;
or 100 ft. 84 in.
59 ft. 7′ ;
or 59 ft. 1008 in.

263. . . . .2832 ; .3125

264. . . . 3.44902

265. . . . 1.061208

266. 490000

267. . . . .39375 ;
.5384375 ;

po.	yds.	ft.	in.
13	2	1	4

	yds.	in.	£	s.	d.
268. . . .	520	3 ;	5	4	2

269. . . . $2\frac{5}{12}$; 0 ;
$283\frac{14}{43}$; $46412\frac{39}{133}$

Ex. Ans.

	£	s.	d.
270. . . .	4	15	$7\frac{3}{11}$
271. . . .	6392	19	$0\frac{3}{4}$;
	178	13	$11\frac{3}{4} + \frac{1}{4}$

272. . . . 1

273. 19752 ; 1.30384048 ;
.612372436

	£	s.	d.
274. . . .	431	0	$9\frac{1}{2} + \frac{2}{5}$

275. . . . 945 ; 2.88449 &c.

276. . . .

	£	s.	d.
A's =	26	8	$1\frac{3}{4} + \frac{1}{41}$;
B's =	79	4	$5\frac{1}{4} + \frac{3}{41}$;
C's =	165	0	$10\frac{3}{4} + \frac{27}{41}$

277. . . .

A's =	276	16	$3\frac{3}{4} + \frac{33}{49}$;
B's =	432	10	$6 + \frac{24}{49}$;
C's =	138	8	$1\frac{3}{4} + \frac{41}{49}$

	mo.	dys.
278. . . .	6	$12\frac{6}{17}$

	£	s.	d.
279. . . .	0	5	2
280. . .	53	2	$5\frac{1}{4} + \frac{418}{509}$
281. . .	10	13	$10\frac{1}{4} + \frac{35}{101}$

282. . $\frac{1}{81}$; .00000144 ;
1.030301

283. . . . .1263 ; 25.68 ;
$2\frac{3}{4} = 2.75$

Ex.	Ans.
284. . . .	37587 p.; 181 lbs. 2 oz.; 31556931 sec.;
	yds. ft. in. 16 20 1456
	£ s. d.
285. . . .	91 15 0;
	231 2 $0\frac{3}{4}$
286. . . .	$12\frac{8}{13}$ miles.
287. . . .	$96\frac{3}{16}$
	lbs. oz.
288. . . .	2 $12\frac{9}{14}$
289. . . .	5 $5\frac{1}{3}$
290. . . .	$\frac{540}{900}$; $\frac{630}{900}$; $\frac{216}{900}$; $\frac{1330}{900}$; $\frac{260}{900}$; $\frac{345}{900}$
291. . . .	£83 6s. $5\frac{1}{4}$d.
292. . . .	1.061208
293. . . .	5 yds. 15 ft.
294. . . .	100 miles.
	£ s. d.
295. . . .	519 17 $10\frac{38}{73}$
	Including only one of the days mentioned.
296. . . .	57 17 $7\frac{1}{2}$
297. . . .	1 7 6
298. . . .	20 0 0
299. . . .	15 : 16; $795\frac{25}{32}$
300. . . .	1; 125510

Ex.	Ans.
301. .	$\frac{540}{900}$; $\frac{630}{900}$; $\frac{216}{900}$; $\frac{330}{900}$; $\frac{260}{900}$; $\frac{345}{900}$
302. . . .	$18\frac{11}{20}$; $13\frac{8}{5}$
303. . . .	$\frac{11}{40}$
	£ s. d.
304. . . .	21 8 $1\frac{1}{2}$
305. . . .	217 7 6;
	103 7 $4\frac{1}{2}$
306. . . .	104043
	£ s. d.
307. . . .	387 1 $1\frac{1}{2}$
308. . . .	21 19 $3\frac{3}{4}$
309. . . .	488
310. . . .	27750; $147\frac{1}{2}$; 4 yds. 7 ft. 1280 in.
311. . . .	54 yards.
312. . . .	15s. $6\frac{3}{4}$d.
313. . . .	17696; 83 guineas 2s. $1\frac{1}{4}$d.; 1647828 lbs.; 2401; 178 days; 17 yards.
314. . . .	£6 1s. 4d.
315. . . .	20 days.
316. . . .	57 yards.
317. . . .	$2\frac{1}{4}$ lbs.
318. . . .	35392 pence; 17 guineas 9s. $6\frac{3}{4}$d.;

Ex.	Ans.
	823914 lbs.;
13 wk. 2 dys. 3 hr. 4 min.;	3 yards.
319. . . .	£12 2*s.* 8*d.*
320. . . .	114 yards.
321. . . .	3 oz.
322. . . .	$\frac{101}{102}$; 2; £4 11*s.* $0\frac{1}{4}d.$
323. . . .	$23\frac{7}{15}$; $\frac{121}{168}$
324. . . .	240 times.
	£ *s.* *d.*
325. . . .	1 1 0
326. . . .	1841 7 $9\frac{1}{4}$; 81 12 $6\frac{1}{10}$
327. . . .	$\frac{245}{8}$
328. .	9876; 27.602536; .019598
329.	6031; 2.223980 &c.
330. . . .	16
331. . . .	.5625; .71428; .03125; $\frac{27303}{106}$; $\frac{1001}{2000}$
332. . . .	16.83375; 1820.9855; 1.5; .0013014; .00015625; $\frac{7118580}{}$
333. . . .	4 men.
334. . . .	$\frac{101}{102}$

Ex.	Ans.
335. . . .	$\frac{121}{168}$
336. . . .	3.0944 days.
337. . .	2457.7565873 &c.
338. . . .	55 min. $6\frac{6}{49}$ sec.
339. . . .	£389 11*s.* $4\frac{1}{4}d.$
340. . . .	8 tons; £774
341. . . .	$7\frac{0}{13}$ per cwt.
342. . . .	4*d.*
343. . . .	£638 15*s.* 6*d.*
344. . . .	2.223980 &c.
345. . .	$\frac{3}{16}$; .006640625
346. . . .	.384615; 4.74194; 3.021975; 13 ac. 1 roo. 14 p.; .002
347. . . .	4 feet.
348. . . .	$5\frac{7}{234}$ feet.
	£ *s.* *d.*
349. . . .	22 13 $2\frac{2}{9}$
350. . . .	18 15 0
351. . . .	£2130$\frac{520}{657}$
352. . . .	3*s.* $11\frac{3}{16}d.$
353. . . .	£5 15*s.* $2\frac{38}{41}d.$
	£
354. . . .	*A*, 22,500; *B*, 15,000; *C*, 7,500
355. . . .	To *A*, 750; *B*, 400

Ex.	Ans.
	s. d.
356. . . .	3 $10\frac{3}{4} + \frac{1}{3\,3}$
357. . . .	$3\frac{10}{19}$ months.
358. . . .	3683.625 fr.
359. . . .	£97 4s. $5\frac{1}{3}$d.
360. . . .	15s. 9d. ; .5
361. . . .	2 ; $\frac{1}{5}$
	£ s. d.
362. . . .	4 11 $0\frac{1}{2}$
363. . . .	3 14 10
364. . . .	$\frac{37}{80}$
365. . . .	42 times.
	£ s. d.
366. . . .	4 16 0
367. . . .	1459 17 $0\frac{3}{4}$
	83 6 $5\frac{1}{2}$
368. . . .	3415 yards.
369. . . .	£49 7s. $8\frac{1}{2}$d.
370. . . .	L's = 875 ; M's = 1680 ; N's = 945
371. . . .	$8\frac{293}{350}$ mo.
372. . . .	$19105\frac{13}{17}$ lire.
373. . . .	2 ; $\frac{5}{9}$; .000048884 ; 1160 ; $\frac{37}{80}$

Ex.	Ans.
374. . . .	216 feet.
	tons. cwt. qr. lbs.
	3 9 1 20
	£194 8s. 0d.
375. . . .	$\frac{9}{16}$
376. . . .	$2\frac{4}{23}$
377. . . .	£10 14s. $3\frac{3}{7}$d.
	h. min. sec.
378. . . .	1 5 $27\frac{7}{11}$
379. . . .	$16\frac{1}{4}$ days.
380. . . .	£525 Stock.
381. . . .	$615\frac{9}{16}$
	£ s. d.
382. . . .	5 13 1
383. . . .	
A's =	26 8 $1\frac{3}{4} + \frac{1}{4\,1}$;
B's =	79 4 $5\frac{1}{4} + \frac{3}{4\,1}$;
C's =	165 0 $10\frac{3}{4} + \frac{3\,7}{4\,1}$;
384. . . .	9 mo.
	s. d. f.
385. . . .	1 $11\frac{1}{4} + \frac{17}{23}$
	tons. qrs. lbs.
386. . . .	1 3 16
387. . . .	$\frac{23}{45}$
388. . . .	$\frac{3}{12}$; $\frac{49}{28}$; $\frac{22}{55}$
389. . . .	.5384375
390. . . .	3 ac. 584 yds. ; 10 ac.

Ex.	Ans.
391. . . .	39.25 yards.
	£ s. d.
392. . . .	405 3 $4\frac{1}{2}$
393. . . .	450 0 0
394. . . .	$37\frac{1}{2}$
	th. gr.
395. . . .	1031 5.43 nearly.
	tons. cwt. qrs. lbs.
396. . . .	3 17 2 $26\frac{1}{4}$
397. . . .	£1 1s. 8d.
398. . . .	46.5 &c. feet.
399. . . .	30s.; 15s.; 10s.; 7s. 6d.; 6s.; 5s.
400. . . .	He would pay 12s. 6d. less per annum.

Division of Decimals.

402. . . .	.3117$\dot{9}$1463$\dot{4}$
403. .	10251.1206896 &c.
404. . . .	1.29578705$\dot{0}$
405. . . .	1000
406.	1765.1573292402 &c.
407. . . .	.02159595 &c.

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